

# 61A Lecture 12

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Monday, September 26

# Implementing Dice

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<http://www.math.utah.edu/~pa/Random/Random.html>

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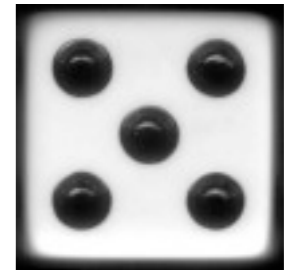
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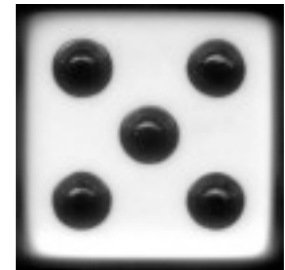
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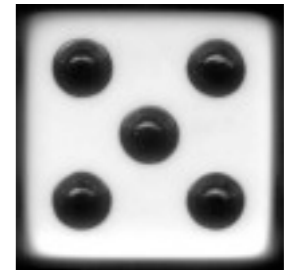
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$$2^{31} - 1$$

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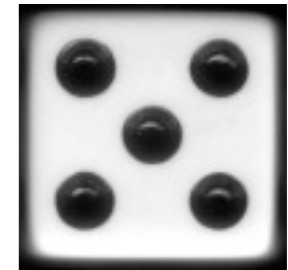
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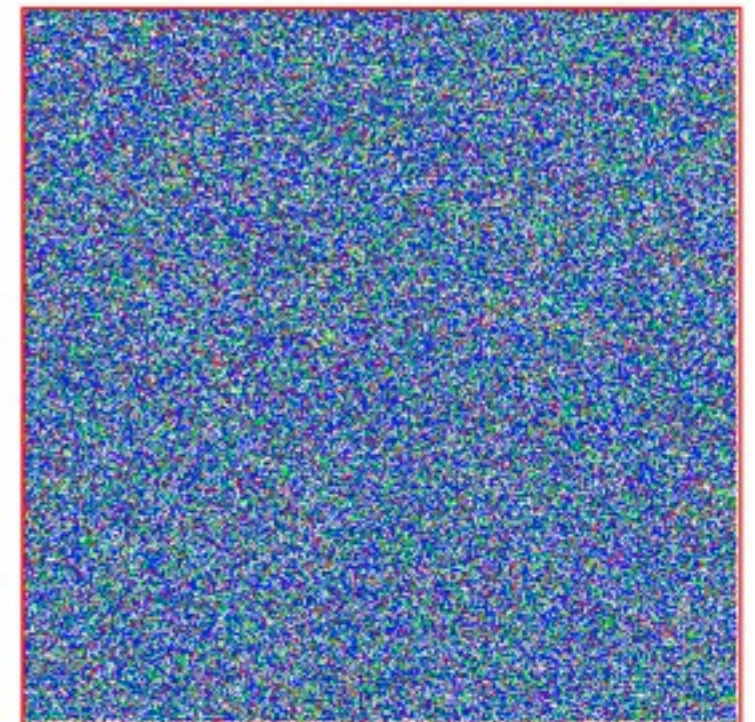
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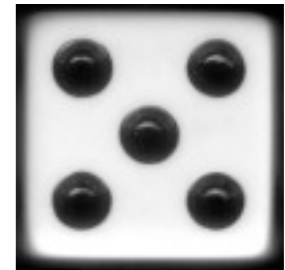
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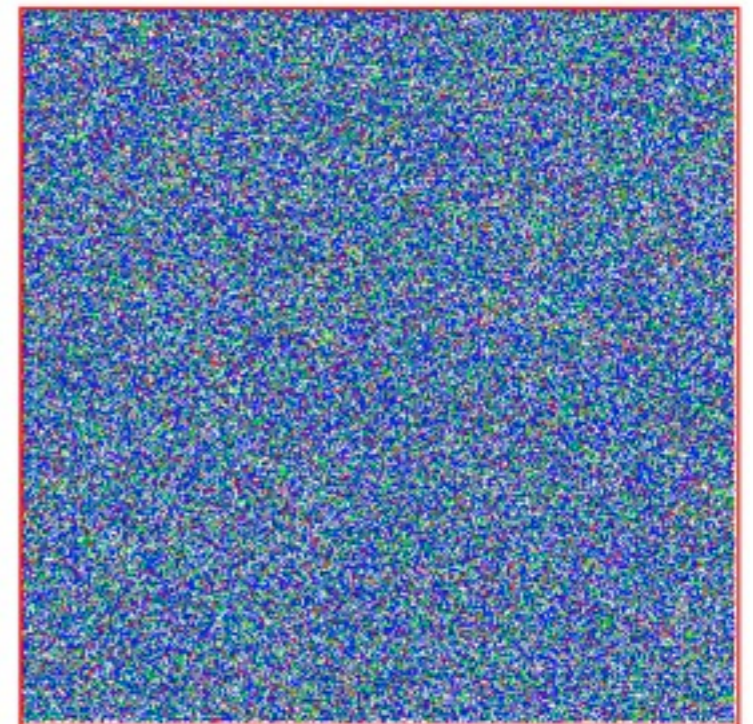
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S.K. Park and K.W. Miller, "Random Number Generators: Good Ones Are Hard To Find", Communications of the ACM, October 1988, pp. 1192-1201.

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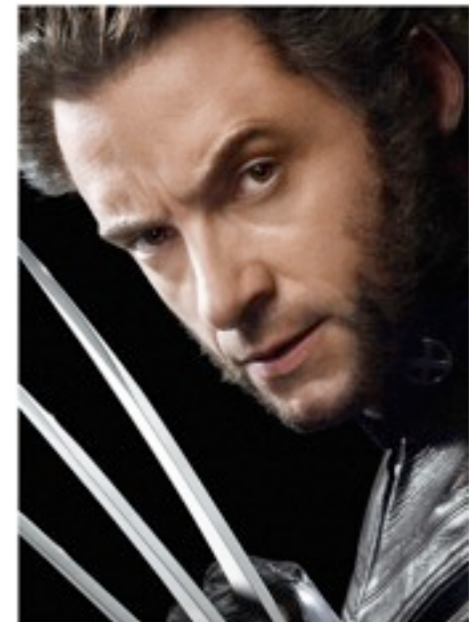
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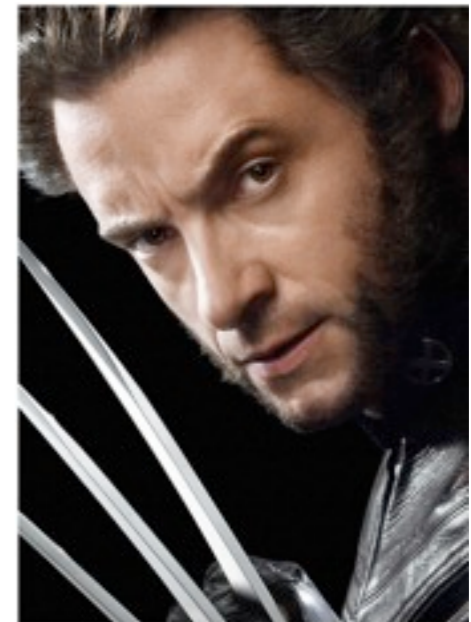
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Demo

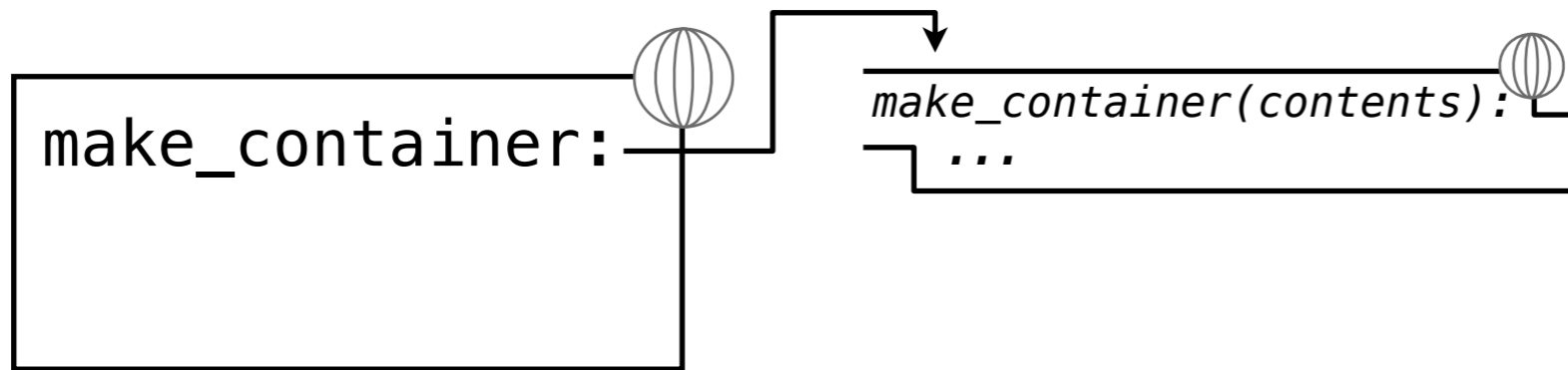
# Implementing a Mutable Container Object

---

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def make_container(contents):  
    def get():  
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    def put(value):  
        nonlocal contents  
        contents = value  
    return get, put  
get, put = make_container('Hi')
```

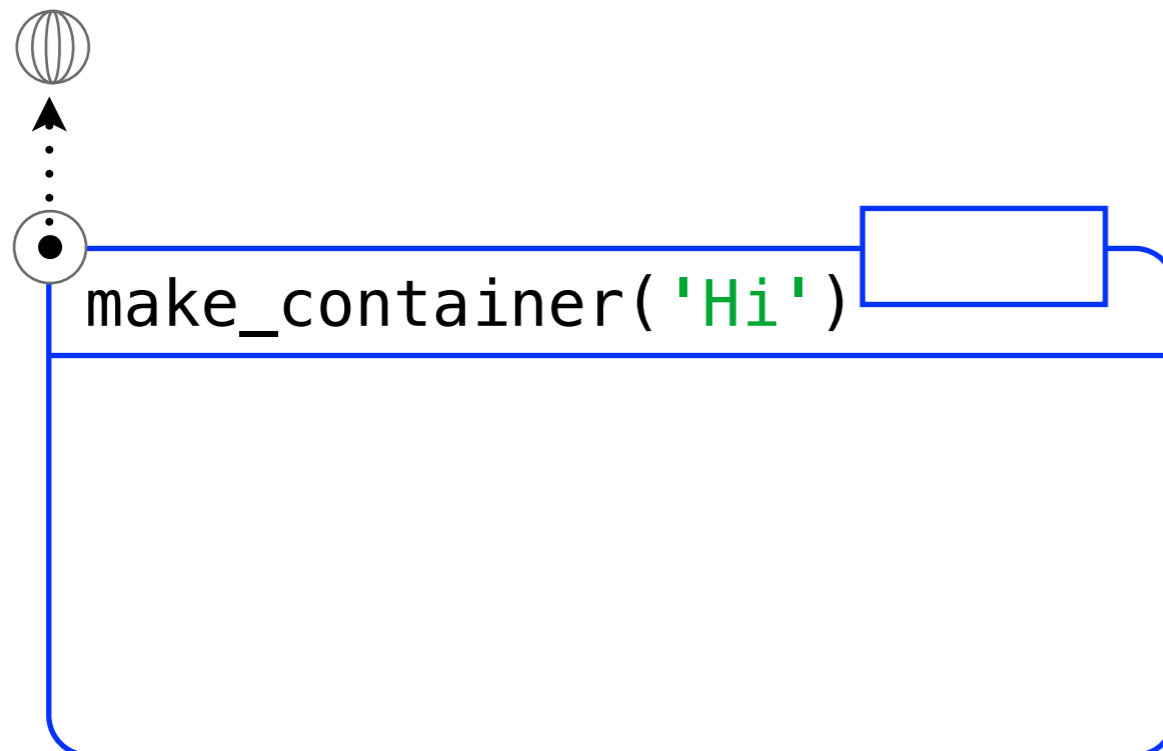
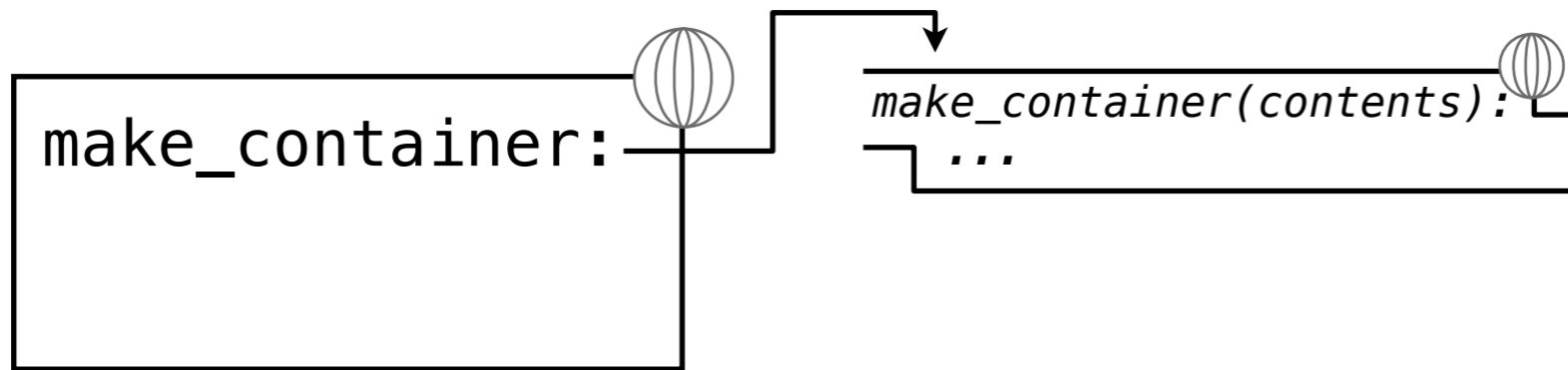
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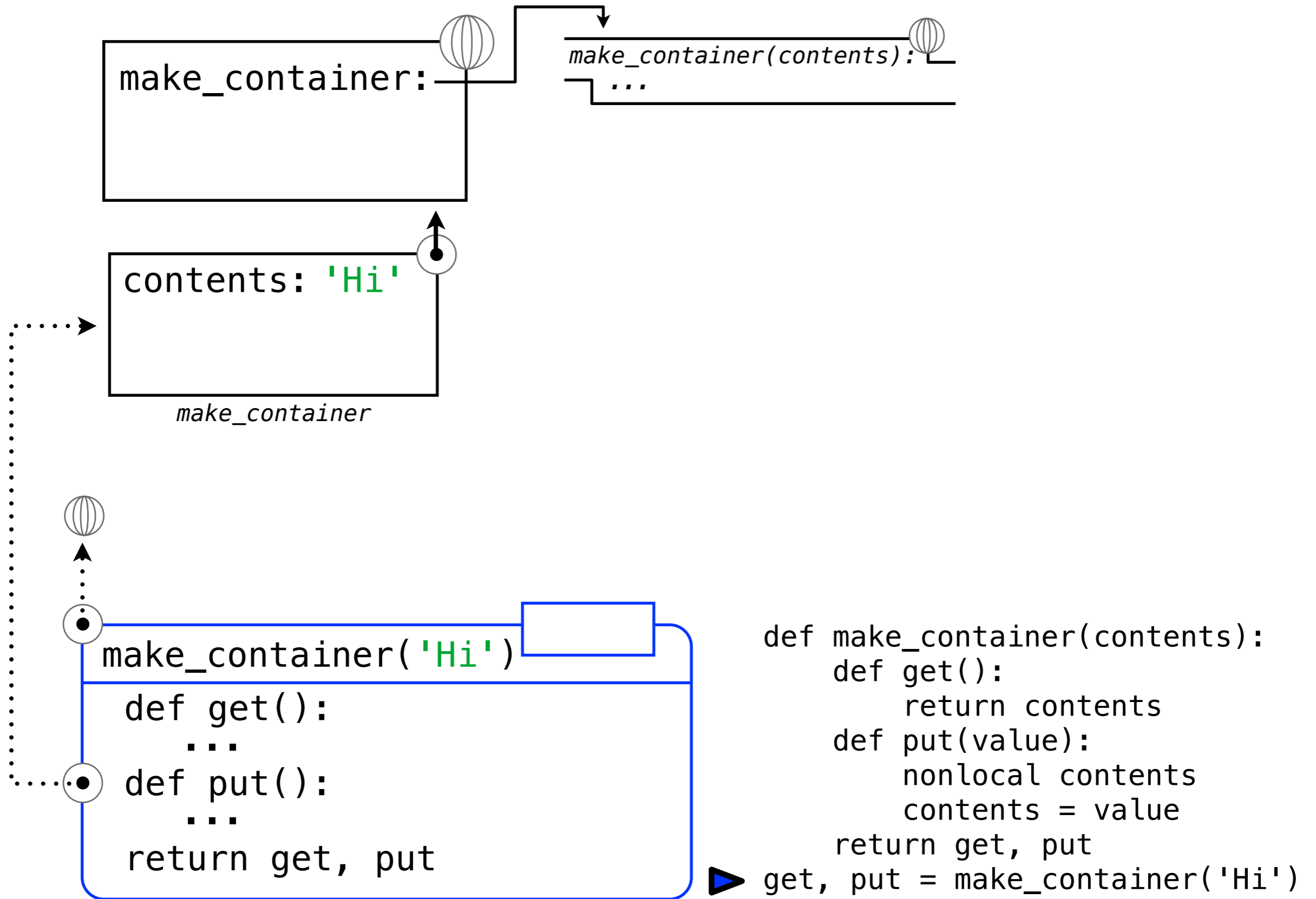
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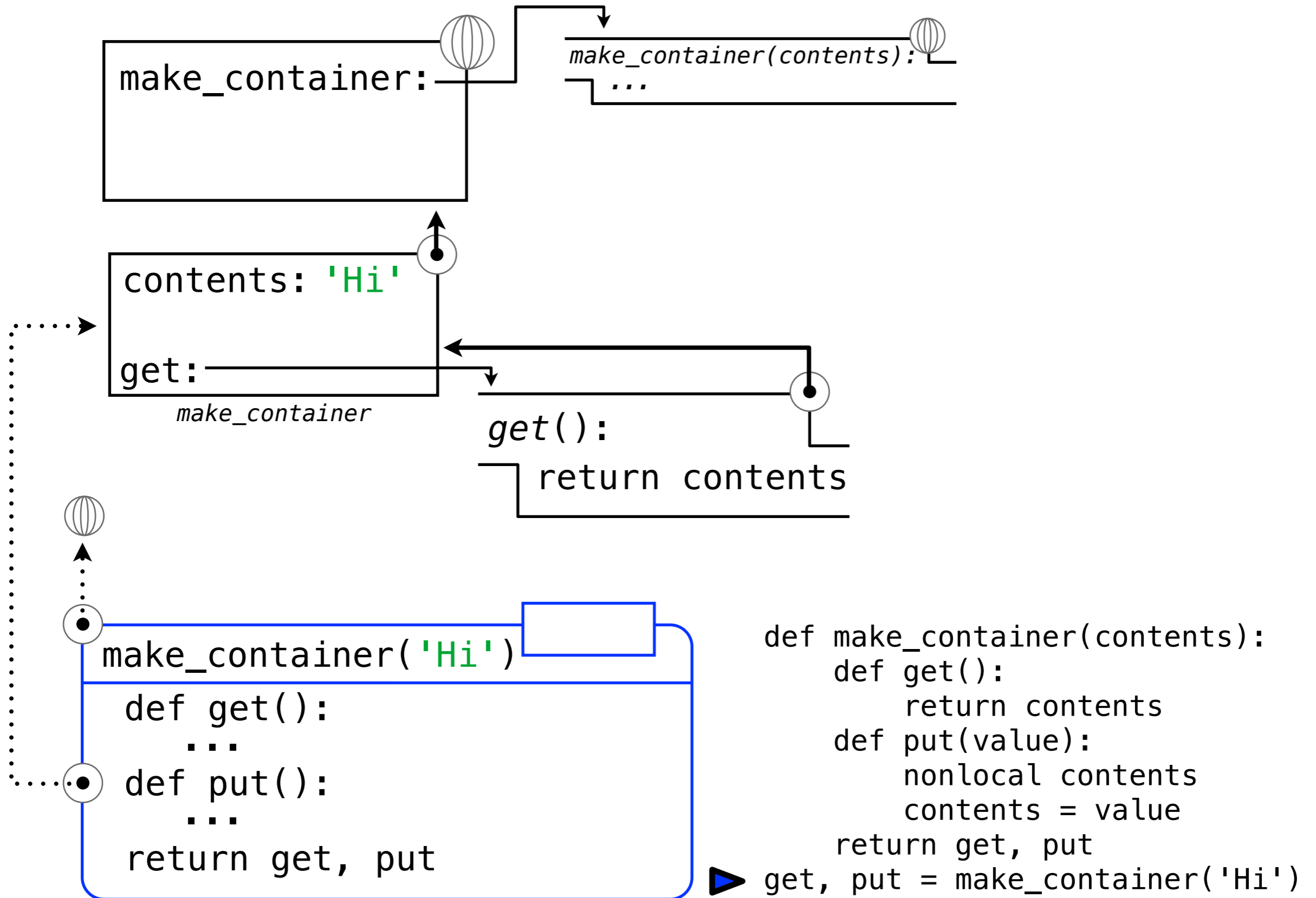
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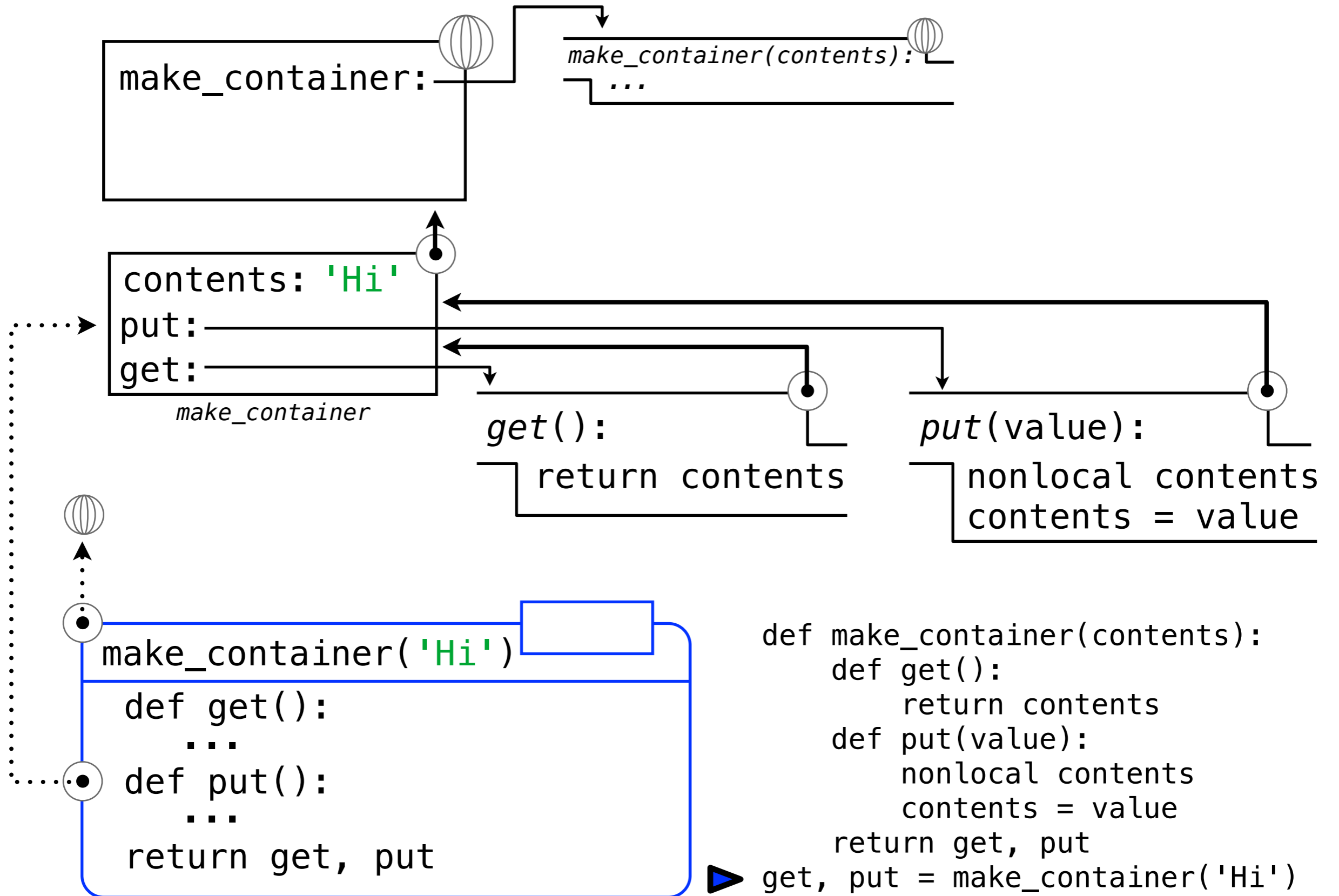




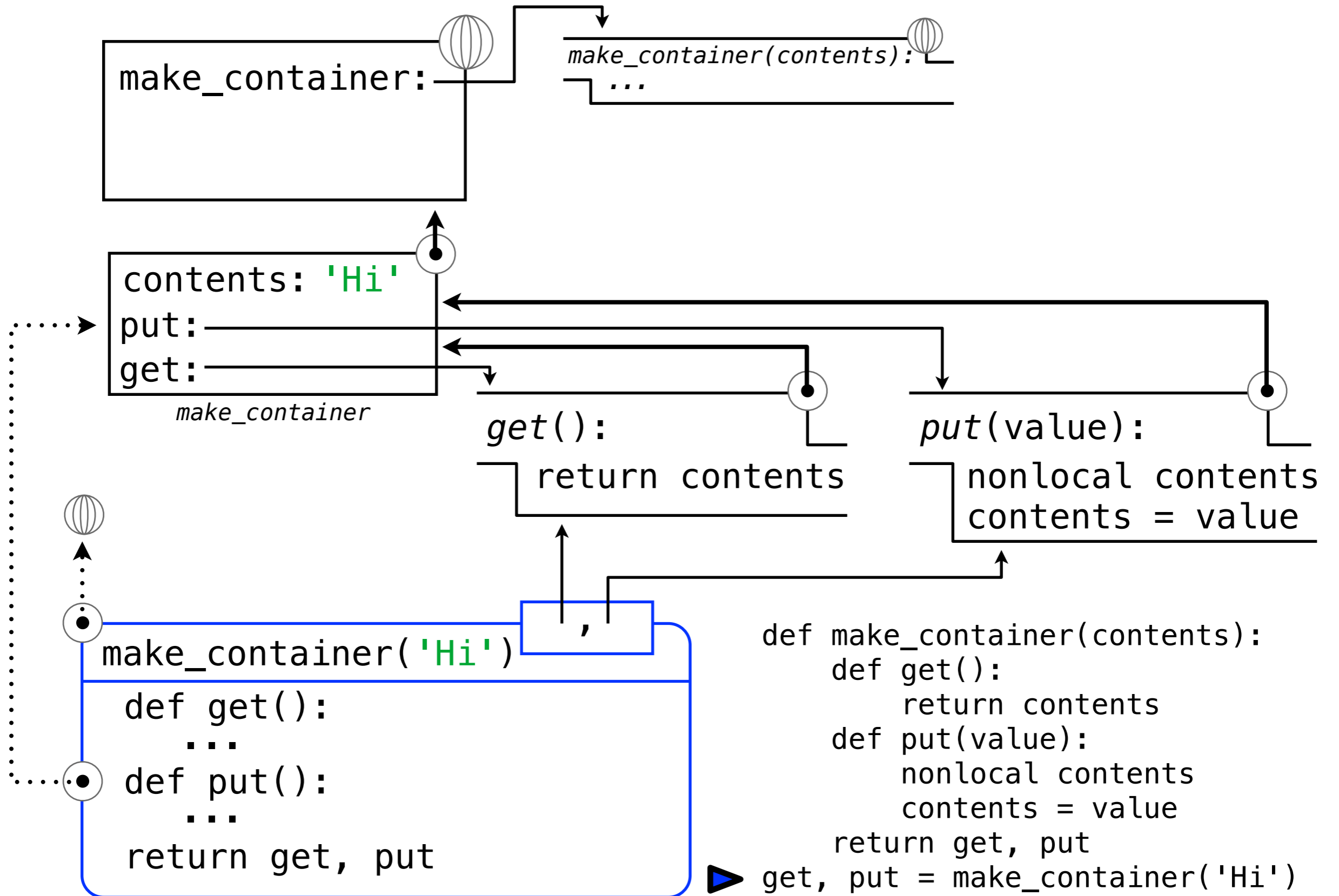
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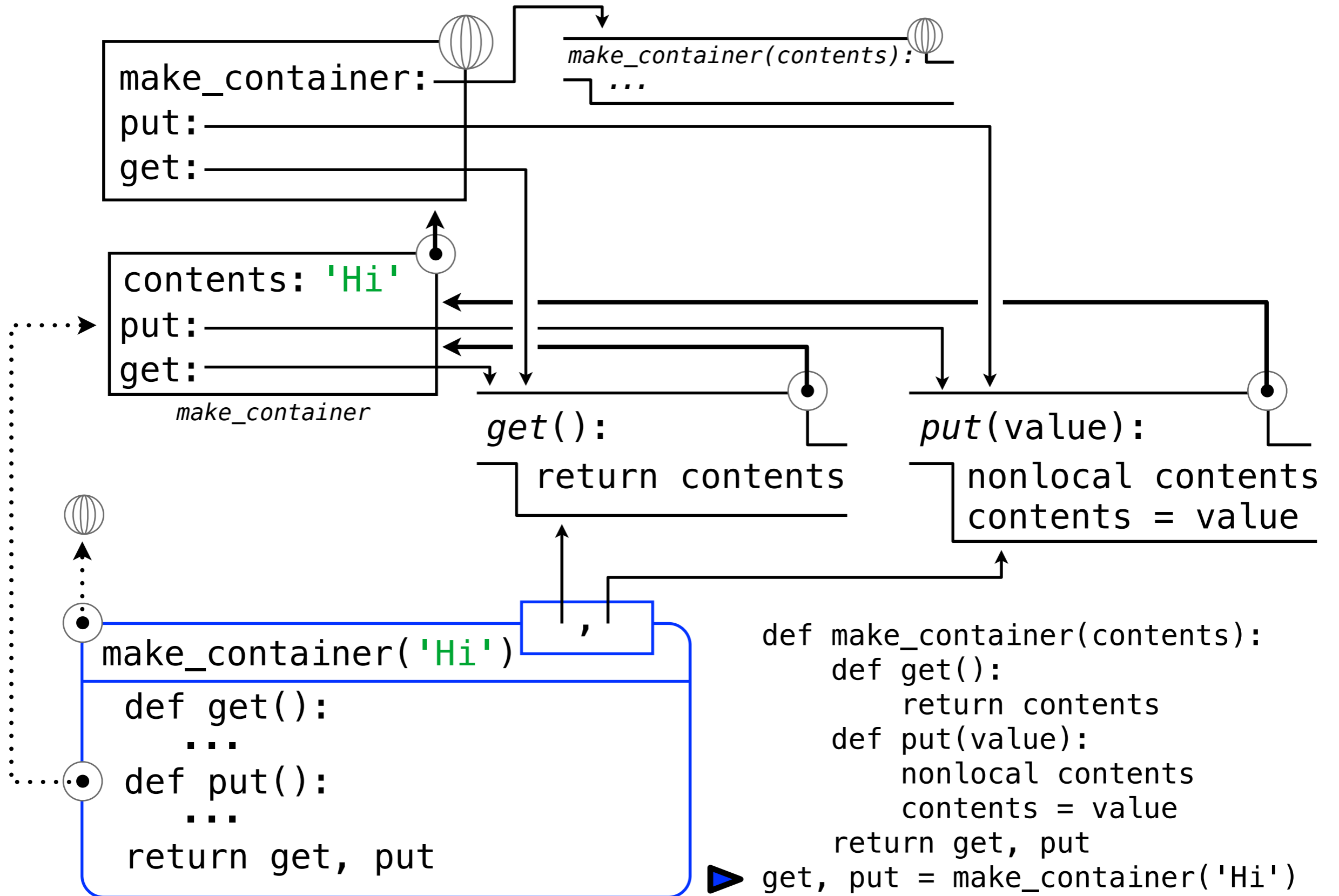
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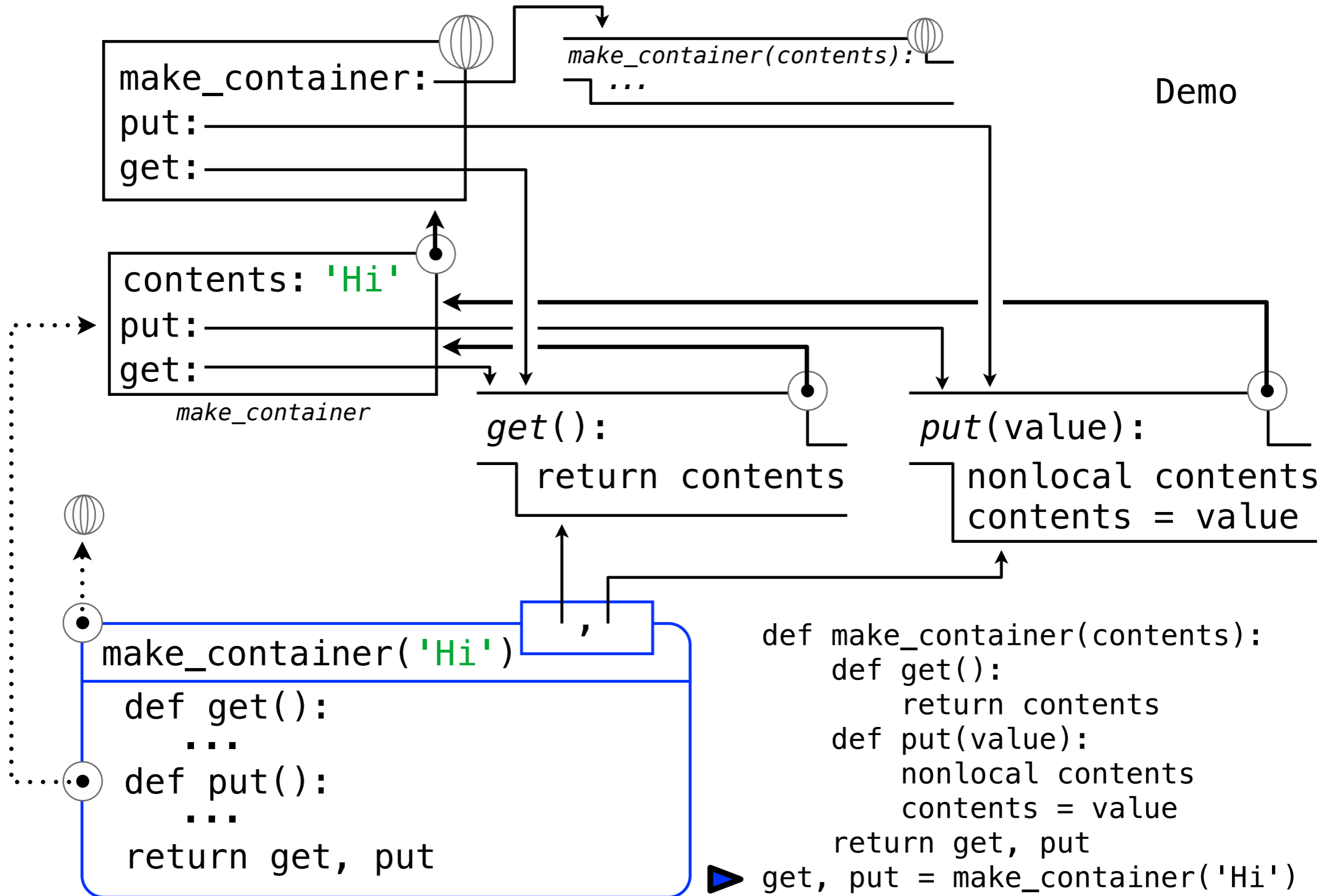
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# Python Lists

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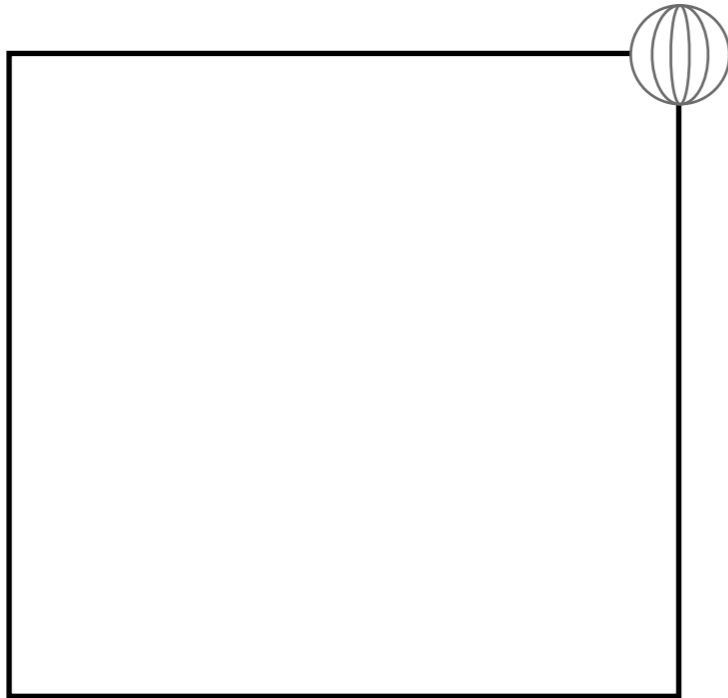
[ 'Demo' ]

<http://docs.python.org/py3k/library/stdtypes.html#mutable-sequence-types>

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# Sharing and Identity with Lists

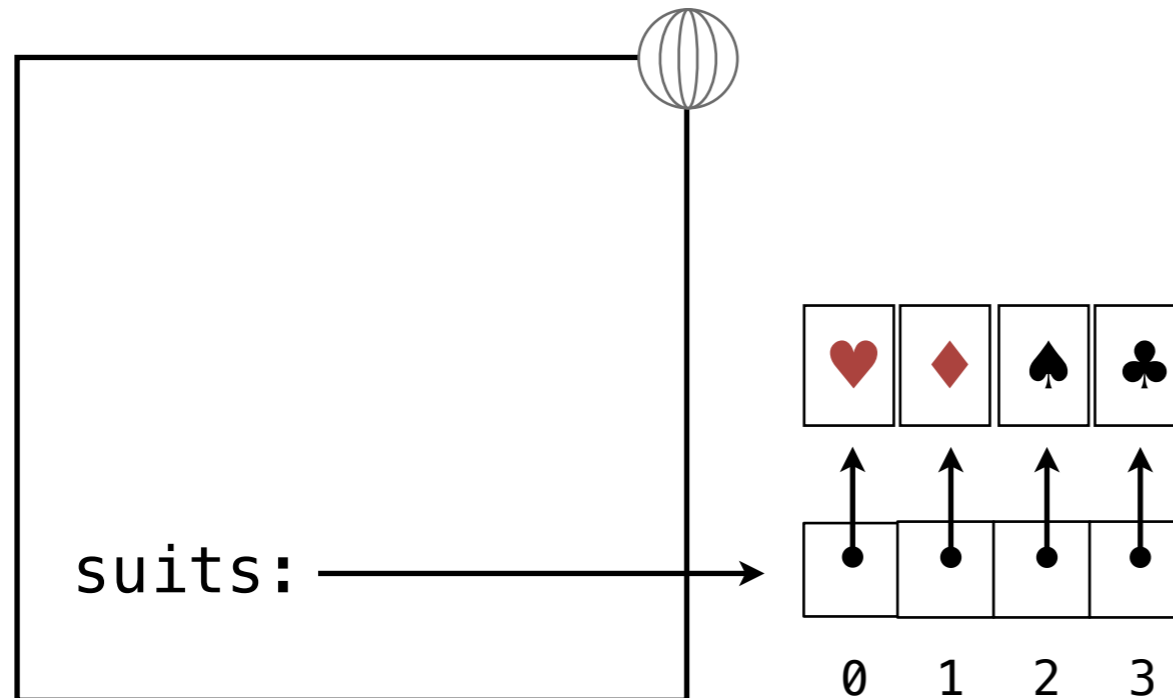
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suits = ['♥', '♦', '♠', '♣']  
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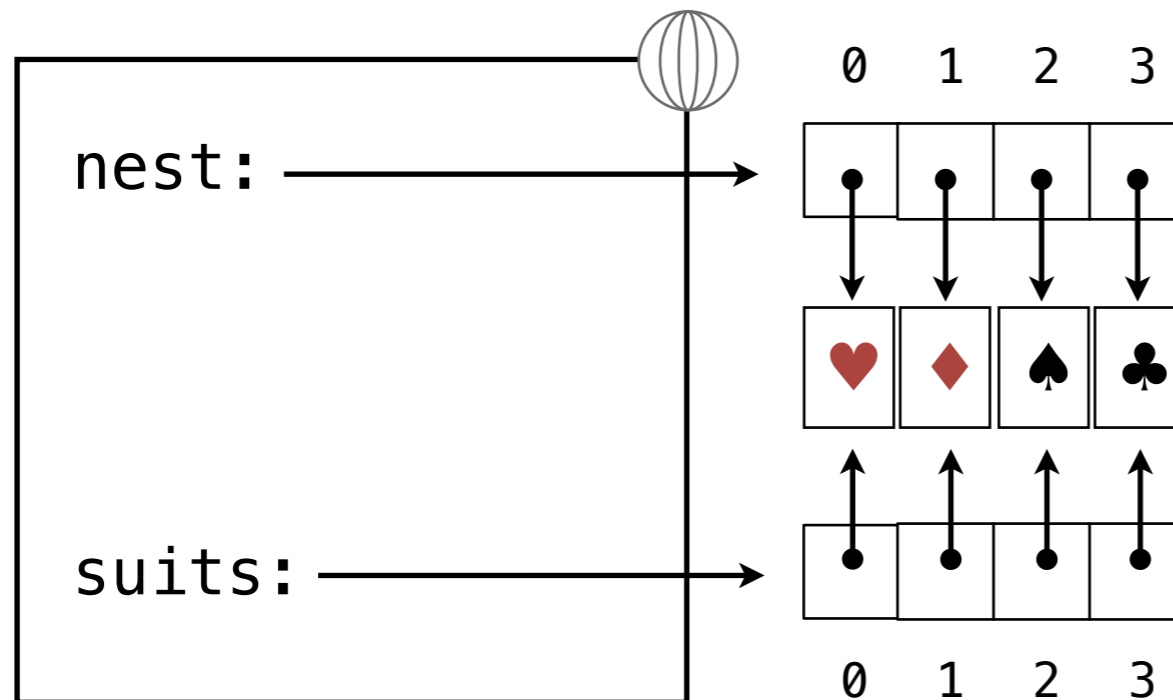
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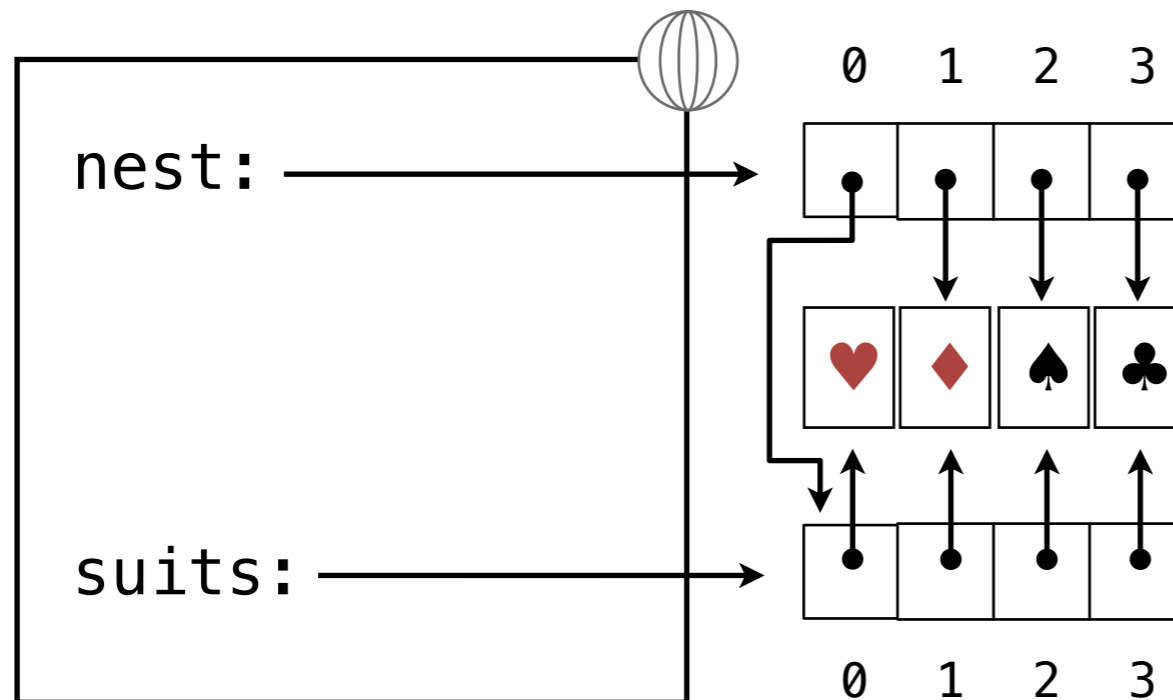


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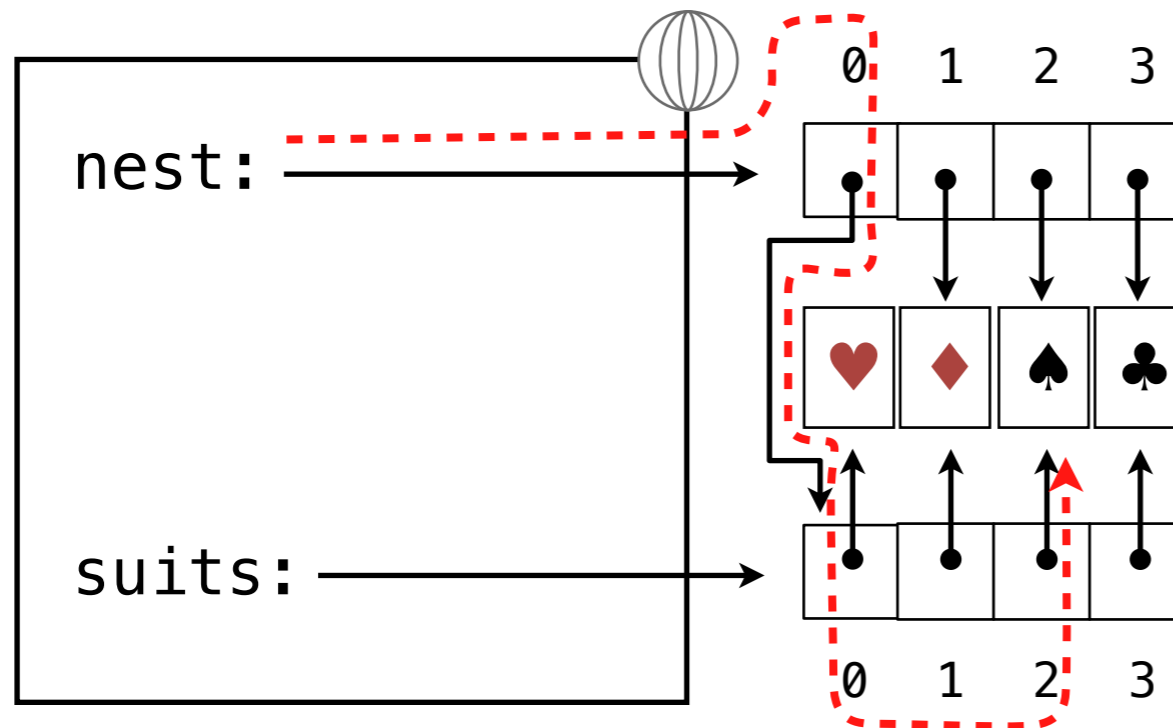
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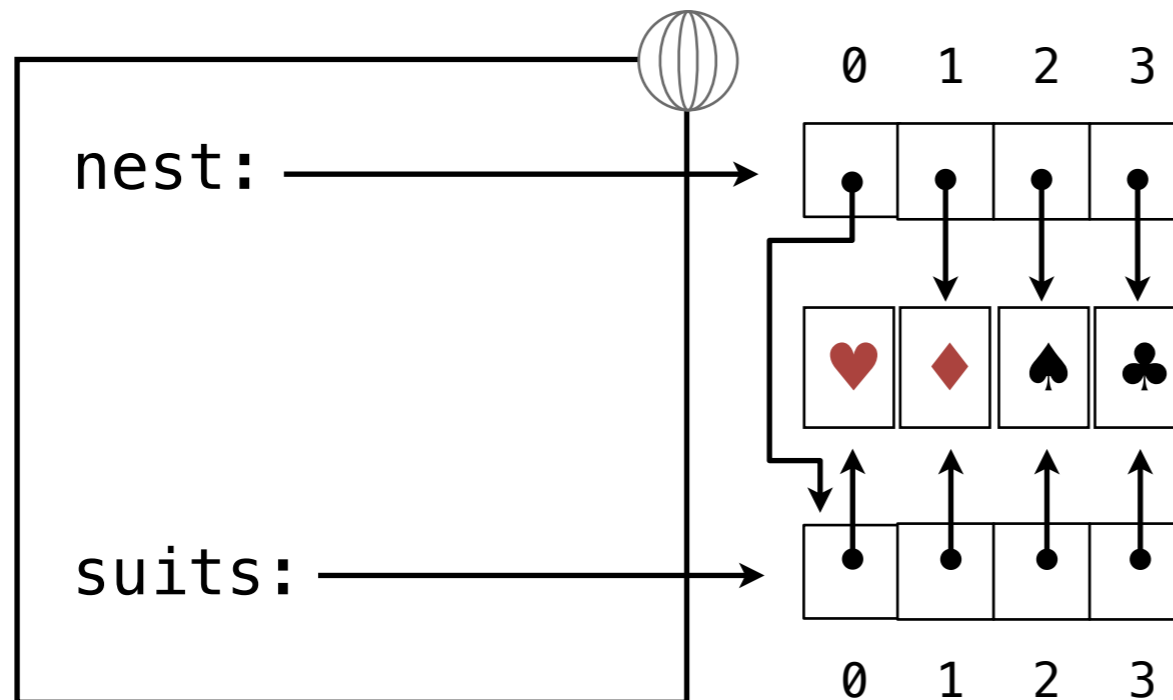
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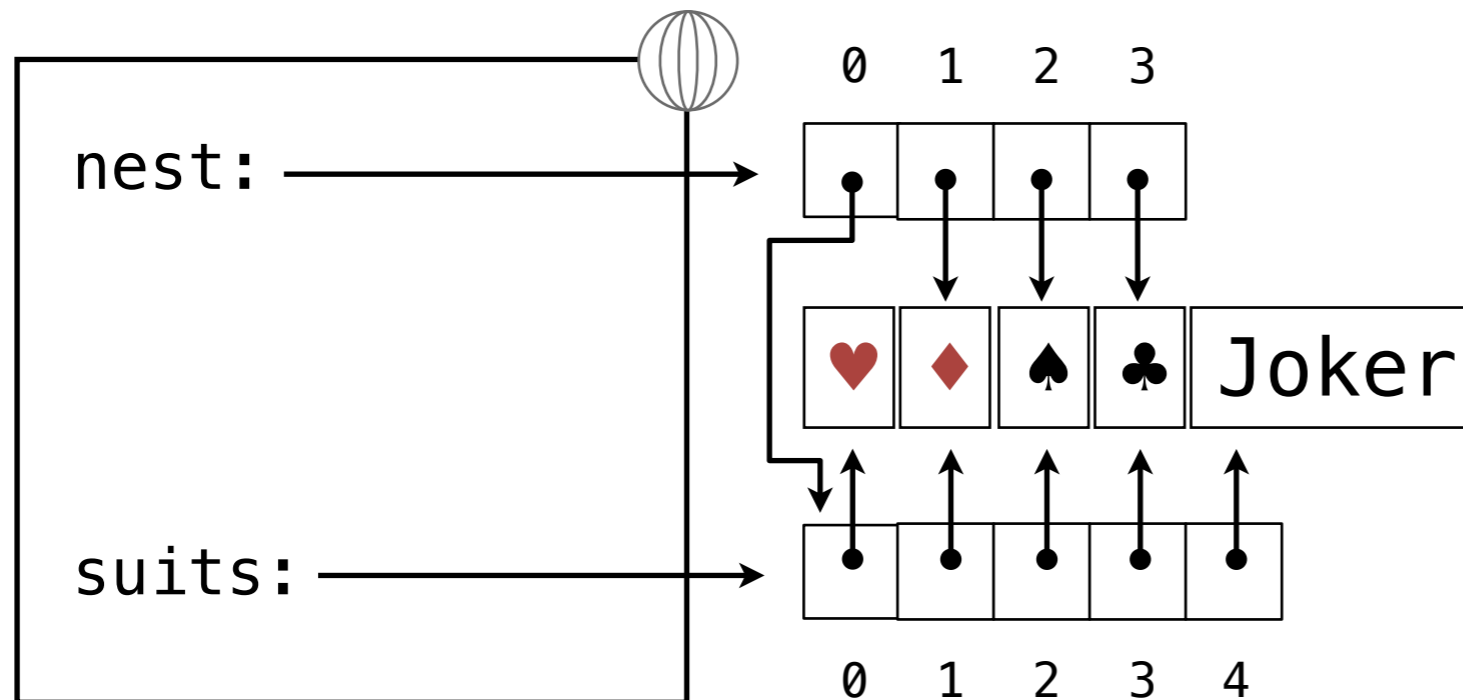
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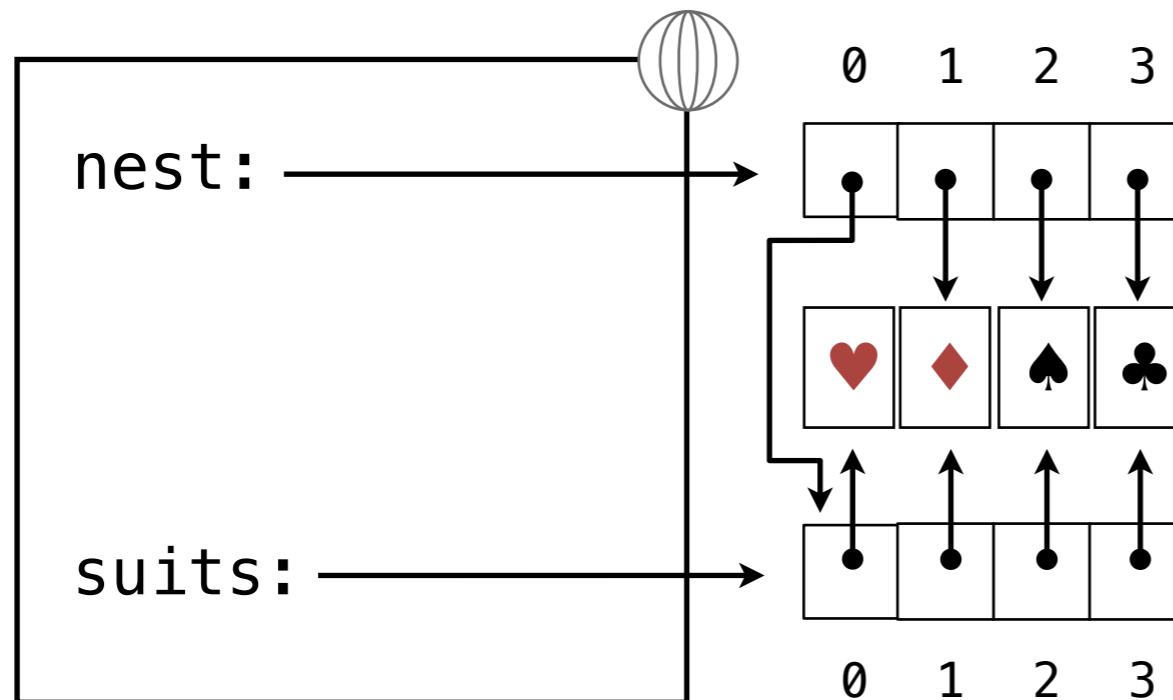
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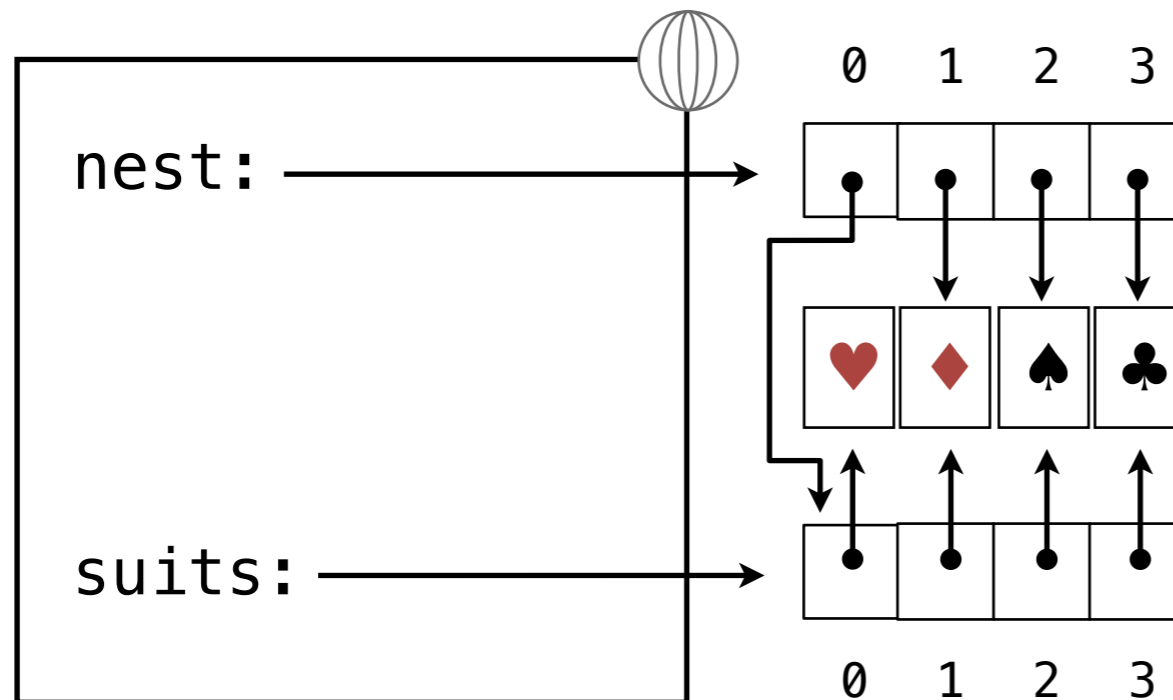
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# Testing for Identity

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```
>>> suits is nest[0]
True
>>> suits is ['♥', '♦', '♠', '♣']
False
>>> suits == ['♥', '♦', '♠', '♣']
True
```

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The body of a dispatch function is always the same:

- One conditional statement with several clauses
- Headers perform equality tests on the message

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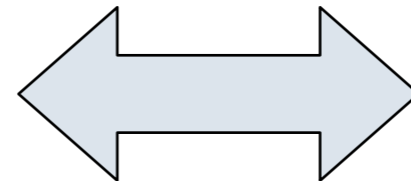
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    def get():
```

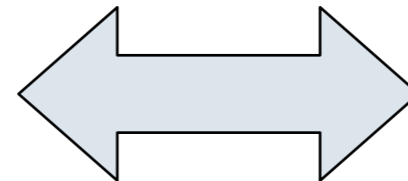
```
        return contents
```

```
    def put(value):
```

```
        nonlocal contents
```

```
        contents = value
```

```
    return get, put
```



Demo

# Implementing Mutable Recursive Lists

---

# Implementing Mutable Recursive Lists

---

Recursive List  
Refresher Demo

# Implementing Mutable Recursive Lists

---

```
def make_mutable_rlist():
```

Recursive List  
Refresher Demo

# Implementing Mutable Recursive Lists

---

```
def make_mutable_rlist():  
    contents = empty_rlist
```

Recursive List  
Refresher Demo

# Implementing Mutable Recursive Lists

---

```
def make_mutable_rlist():  
    contents = empty_rlist  
    def dispatch(message, value=None):
```

Recursive List  
Refresher Demo

# Implementing Mutable Recursive Lists

---

```
def make_mutable_rlist():  
    contents = empty_rlist  
    def dispatch(message, value=None):  
        nonlocal contents
```

Recursive List  
Refresher Demo



# Implementing Mutable Recursive Lists

---

```
def make_mutable_rlist():  
    contents = empty_rlist  
    def dispatch(message, value=None):  
        nonlocal contents  
        if message == 'len':
```

Recursive List  
Refresher Demo

# Implementing Mutable Recursive Lists

---

```
def make_mutable_rlist():
    contents = empty_rlist
    def dispatch(message, value=None):
        nonlocal contents
        if message == 'len':
            return len_rlist(contents)
```

Recursive List  
Refresher Demo

# Implementing Mutable Recursive Lists

---

```
def make_mutable_rlist():
    contents = empty_rlist
    def dispatch(message, value=None):
        nonlocal contents
        if message == 'len':
            return len_rlist(contents)
        elif message == 'getitem':
```

Recursive List  
Refresher Demo

# Implementing Mutable Recursive Lists

---

```
def make_mutable_rlist():  
    contents = empty_rlist  
    def dispatch(message, value=None):  
        nonlocal contents  
        if message == 'len':  
            return len_rlist(contents)  
        elif message == 'getitem':  
            return getitem_rlist(contents, value)
```

Recursive List  
Refresher Demo

# Implementing Mutable Recursive Lists

---

```
def make_mutable_rlist():  
    contents = empty_rlist  
    def dispatch(message, value=None):  
        nonlocal contents  
        if message == 'len':  
            return len_rlist(contents)  
        elif message == 'getitem':  
            return getitem_rlist(contents, value)  
        elif message == 'push_first':
```

Recursive List  
Refresher Demo

# Implementing Mutable Recursive Lists

---

```
def make_mutable_rlist():  
    contents = empty_rlist  
    def dispatch(message, value=None):  
        nonlocal contents  
        if message == 'len':  
            return len_rlist(contents)  
        elif message == 'getitem':  
            return getitem_rlist(contents, value)  
        elif message == 'push_first':  
            contents = make_rlist(value, contents)
```

Recursive List  
Refresher Demo

# Implementing Mutable Recursive Lists

---

```
def make_mutable_rlist():  
    contents = empty_rlist  
    def dispatch(message, value=None):  
        nonlocal contents  
        if message == 'len':  
            return len_rlist(contents)  
        elif message == 'getitem':  
            return getitem_rlist(contents, value)  
        elif message == 'push_first':  
            contents = make_rlist(value, contents)  
        elif message == 'pop_first':
```

Recursive List  
Refresher Demo

# Implementing Mutable Recursive Lists

---

```
def make_mutable_rlist():  
    contents = empty_rlist  
    def dispatch(message, value=None):  
        nonlocal contents  
        if message == 'len':  
            return len_rlist(contents)  
        elif message == 'getitem':  
            return getitem_rlist(contents, value)  
        elif message == 'push_first':  
            contents = make_rlist(value, contents)  
        elif message == 'pop_first':  
            f = first(contents)
```

Recursive List  
Refresher Demo



# Implementing Mutable Recursive Lists

---

```
def make_mutable_rlist():
    contents = empty_rlist
    def dispatch(message, value=None):
        nonlocal contents
        if message == 'len':
            return len_rlist(contents)
        elif message == 'getitem':
            return getitem_rlist(contents, value)
        elif message == 'push_first':
            contents = make_rlist(value, contents)
        elif message == 'pop_first':
            f = first(contents)
            contents = rest(contents)
```

Recursive List  
Refresher Demo

# Implementing Mutable Recursive Lists

---

```
def make_mutable_rlist():
    contents = empty_rlist
    def dispatch(message, value=None):
        nonlocal contents
        if message == 'len':
            return len_rlist(contents)
        elif message == 'getitem':
            return getitem_rlist(contents, value)
        elif message == 'push_first':
            contents = make_rlist(value, contents)
        elif message == 'pop_first':
            f = first(contents)
            contents = rest(contents)
            return f
```

Recursive List  
Refresher Demo

# Implementing Mutable Recursive Lists

---

```
def make_mutable_rlist():
    contents = empty_rlist
    def dispatch(message, value=None):
        nonlocal contents
        if message == 'len':
            return len_rlist(contents)
        elif message == 'getitem':
            return getitem_rlist(contents, value)
        elif message == 'push_first':
            contents = make_rlist(value, contents)
        elif message == 'pop_first':
            f = first(contents)
            contents = rest(contents)
            return f
        elif message == 'str':
```

Recursive List  
Refresher Demo

# Implementing Mutable Recursive Lists

---

```
def make_mutable_rlist():
    contents = empty_rlist
    def dispatch(message, value=None):
        nonlocal contents
        if message == 'len':
            return len_rlist(contents)
        elif message == 'getitem':
            return getitem_rlist(contents, value)
        elif message == 'push_first':
            contents = make_rlist(value, contents)
        elif message == 'pop_first':
            f = first(contents)
            contents = rest(contents)
            return f
        elif message == 'str':
            return str(contents)
```

Recursive List  
Refresher Demo

# Implementing Mutable Recursive Lists

---

```
def make_mutable_rlist():
    contents = empty_rlist
    def dispatch(message, value=None):
        nonlocal contents
        if message == 'len':
            return len_rlist(contents)
        elif message == 'getitem':
            return getitem_rlist(contents, value)
        elif message == 'push_first':
            contents = make_rlist(value, contents)
        elif message == 'pop_first':
            f = first(contents)
            contents = rest(contents)
            return f
        elif message == 'str':
            return str(contents)
    return dispatch
```

Recursive List  
Refresher Demo

# Implementing Mutable Recursive Lists

---

```
def make_mutable_rlist():
    contents = empty_rlist
    def dispatch(message, value=None):
        nonlocal contents
        if message == 'len':
            return len_rlist(contents)
        elif message == 'getitem':
            return getitem_rlist(contents, value)
        elif message == 'push_first':
            contents = make_rlist(value, contents)
        elif message == 'pop_first':
            f = first(contents)
            contents = rest(contents)
            return f
        elif message == 'str':
            return str(contents)
    return dispatch
```

Recursive List  
Refresher Demo

Demo