

# 61A Lecture 13

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Wednesday, September 28

# Dictionaries

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```

# Limitations on Dictionaries

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This first restriction is tied to Python's underlying implementation of dictionaries.

The second restriction is an intentional consequence of the dictionary abstraction.

# Implementing Dictionaries

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        records.append([key, value])
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        if message == 'getitem':
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Question: Do we need a nonlocal statement here?

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Demo



# Message Passing

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An approach to organizing the relationship among different pieces of a program



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An approach to organizing the relationship among different pieces of a program

Different objects pass messages to each other



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Different objects pass messages to each other

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Encapsulates the behavior of all operations on a piece of data within one function that responds to different messages.



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An approach to organizing the relationship among different pieces of a program

Different objects pass messages to each other

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Encapsulates the behavior of all operations on a piece of data within one function that responds to different messages.

Important historical interest: the message passing approach strongly influenced object-oriented programming (next lecture).



# Dispatch Dictionaries

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# Example: Constraint Programming

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$$a + b = c$$

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$$a + b = c$$

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Python functions are *procedural*. They describe how to compute a particular result from a particular set of inputs.

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Constraint programming:

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**Challenge:** We want a general means of combination.

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# A Constraint Network for Temperature Conversion

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Combination idea: All intermediate quantities have values too.

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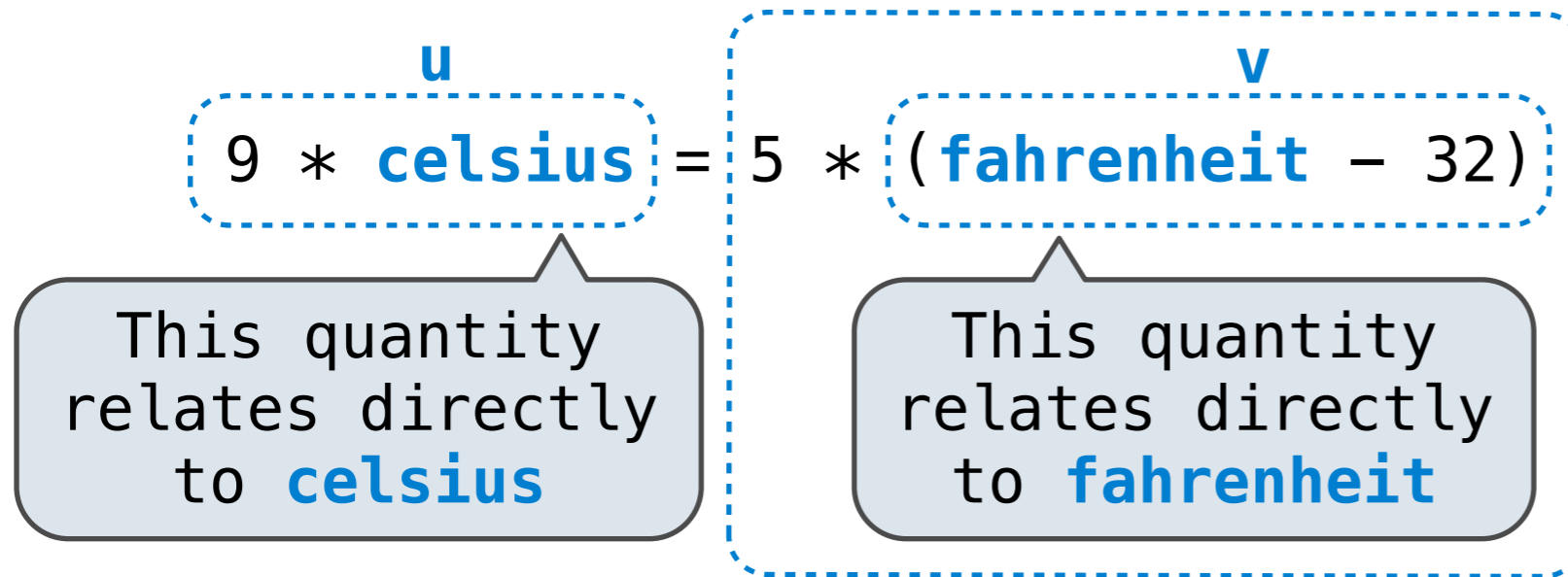
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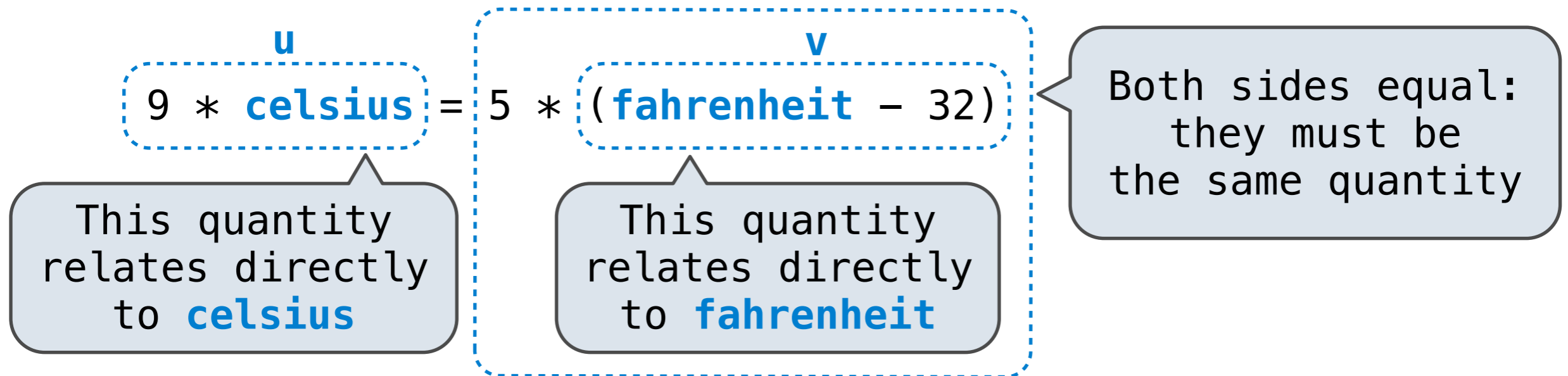
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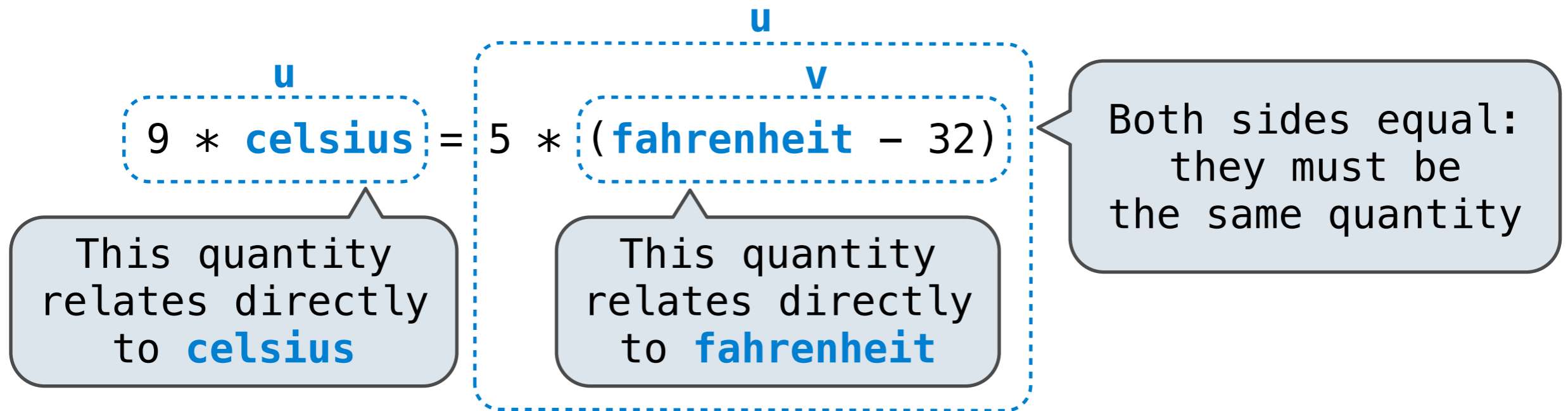
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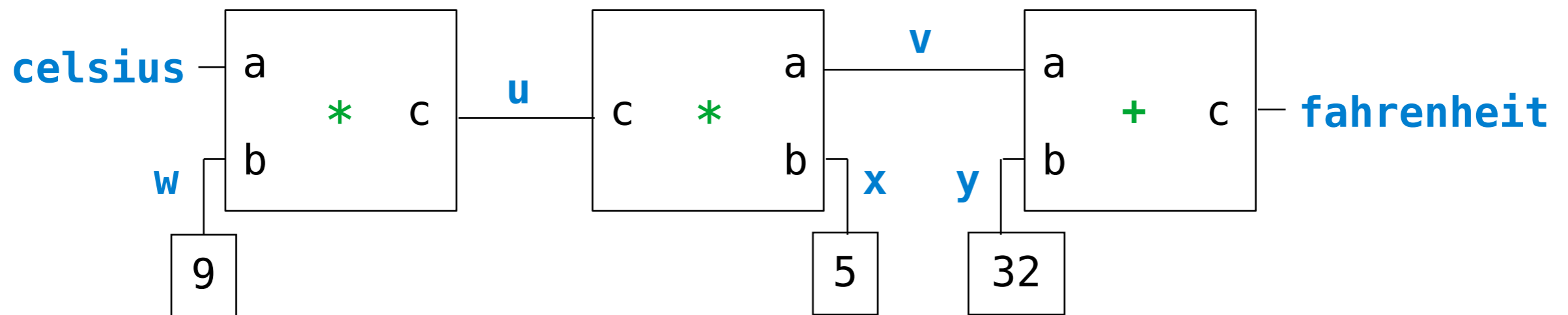
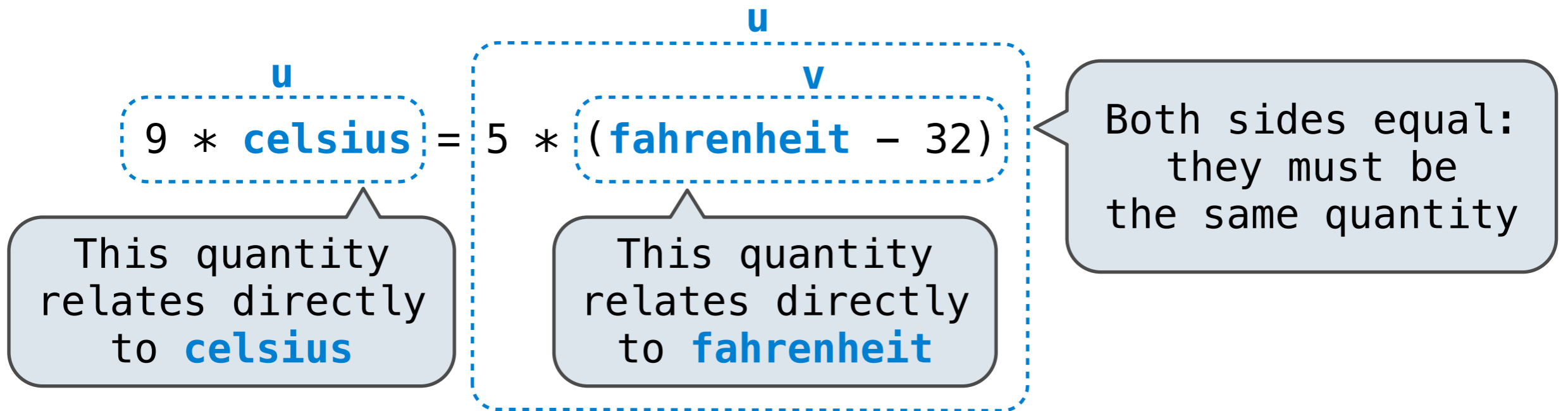
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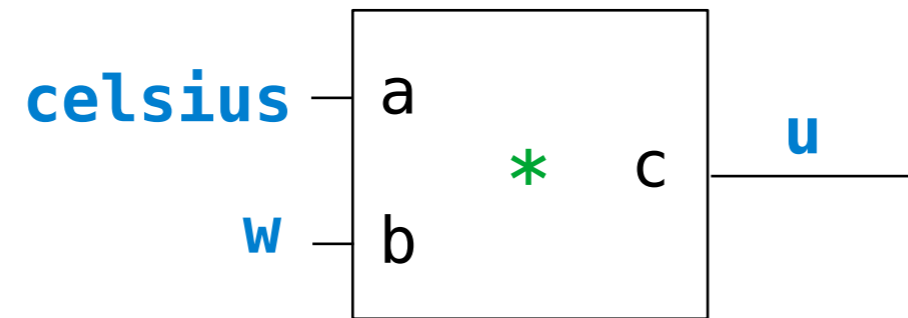
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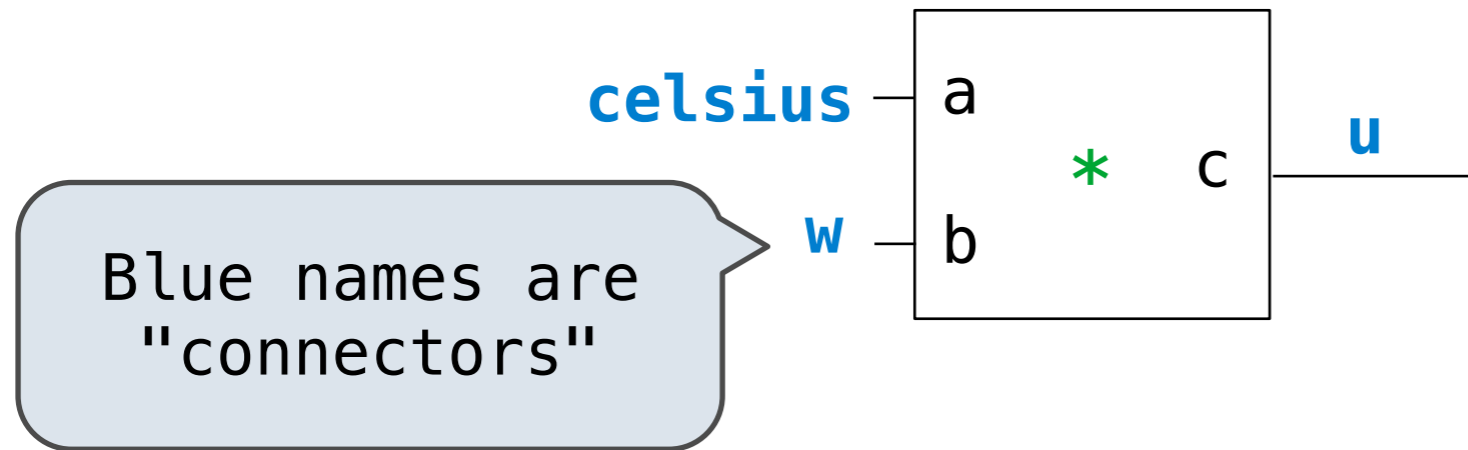
# Anatomy of a Constraint

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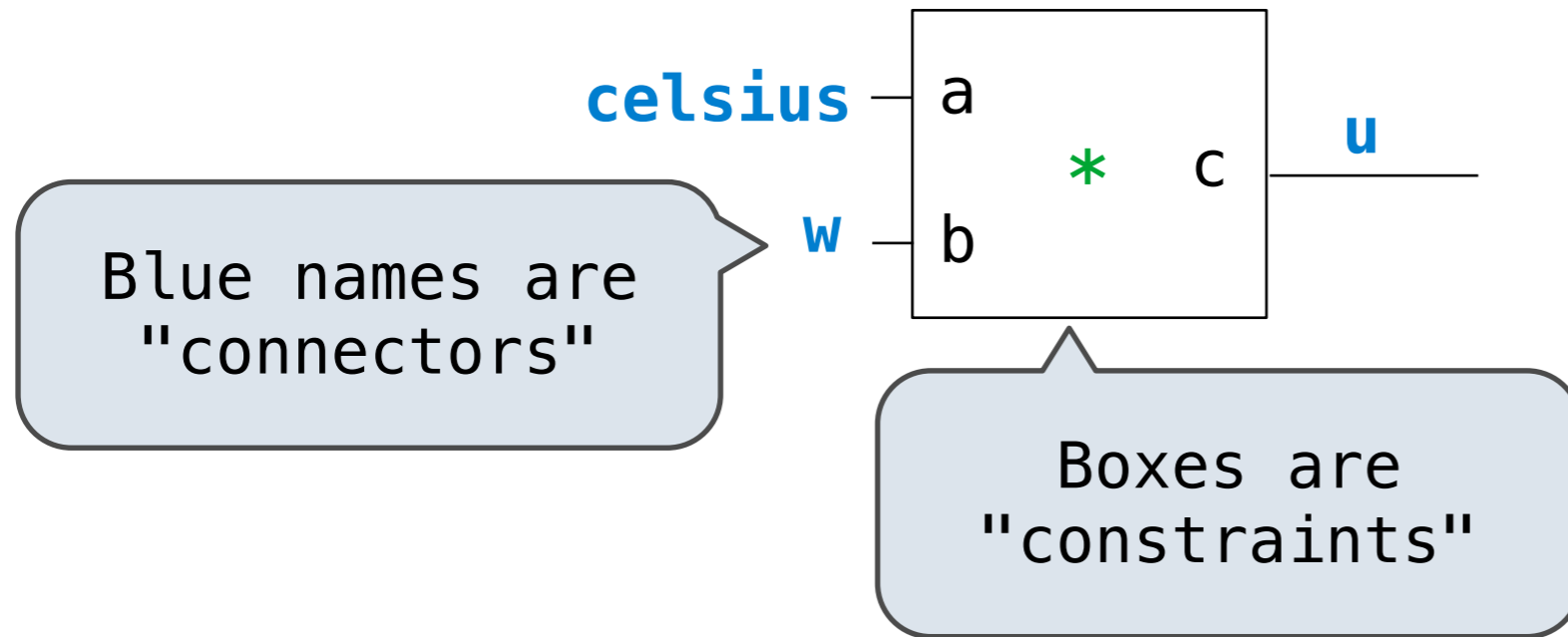
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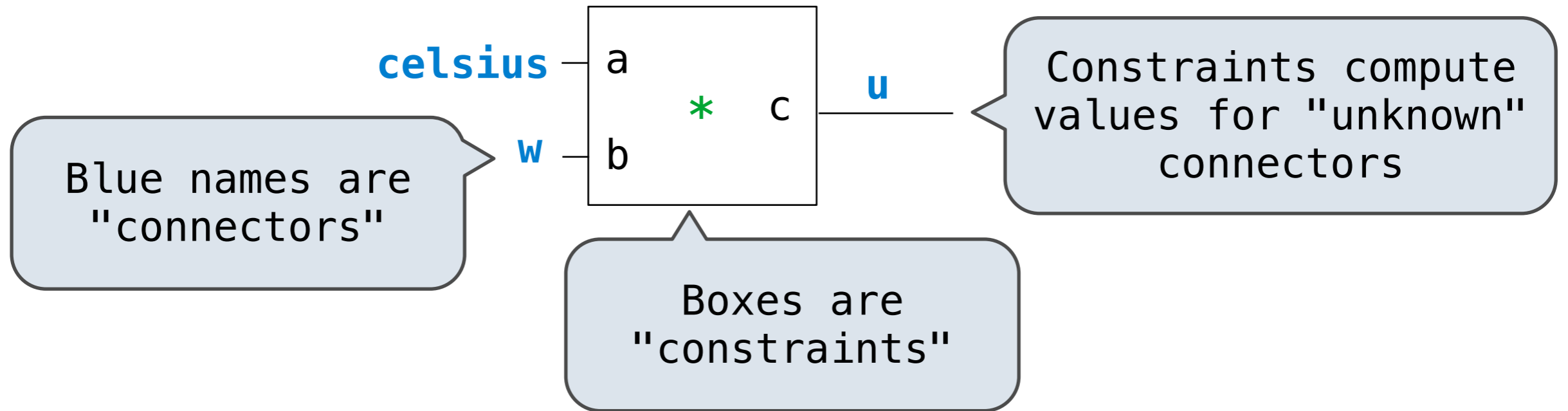
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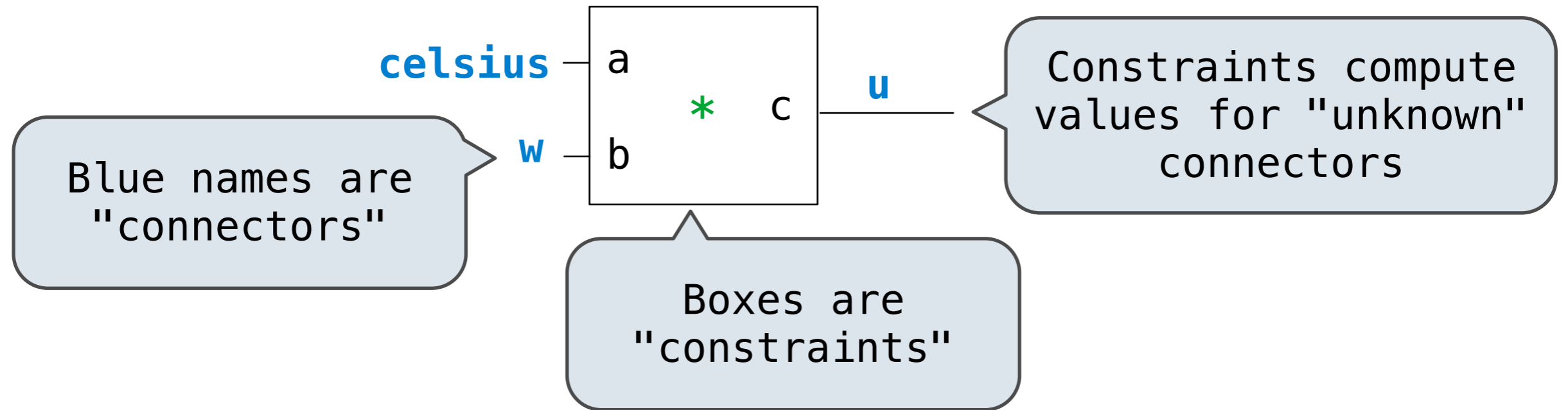
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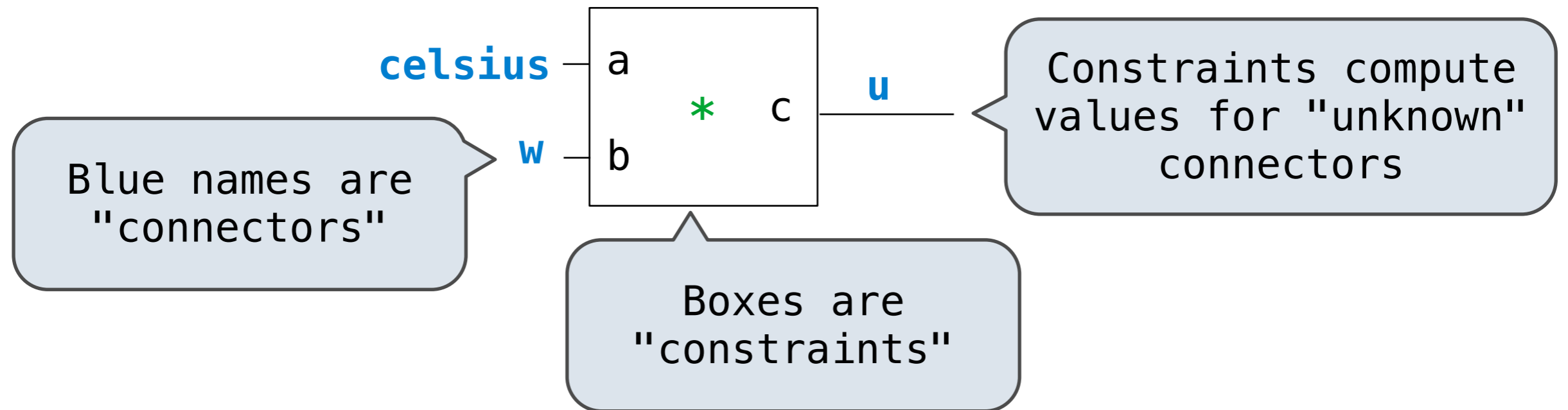
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- *Connectors* represent quantities that have values.

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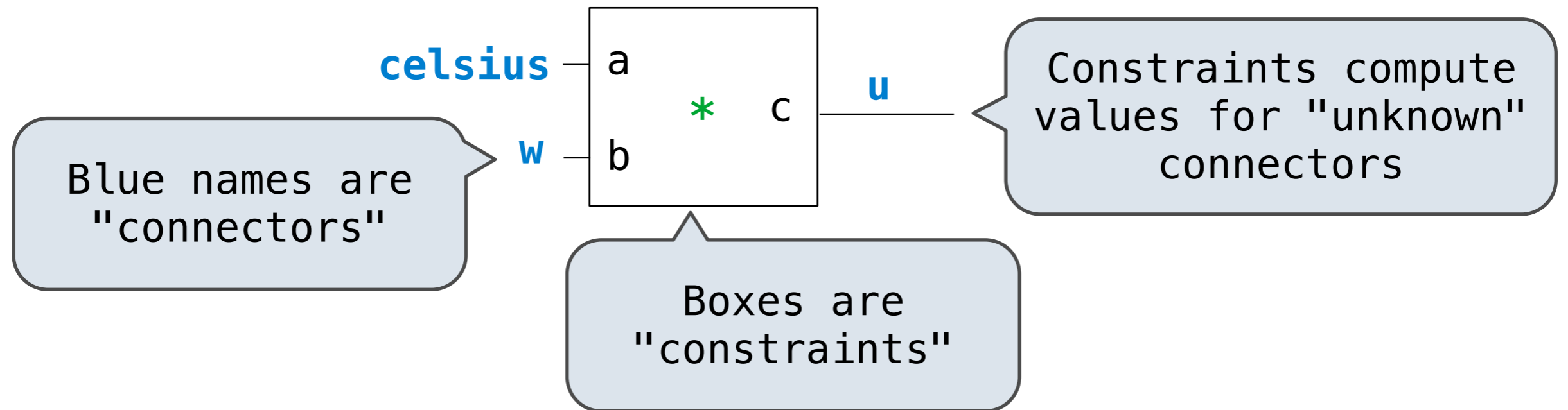
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- *Connectors* represent quantities that have values.
- *Constraints* spread information among connectors.

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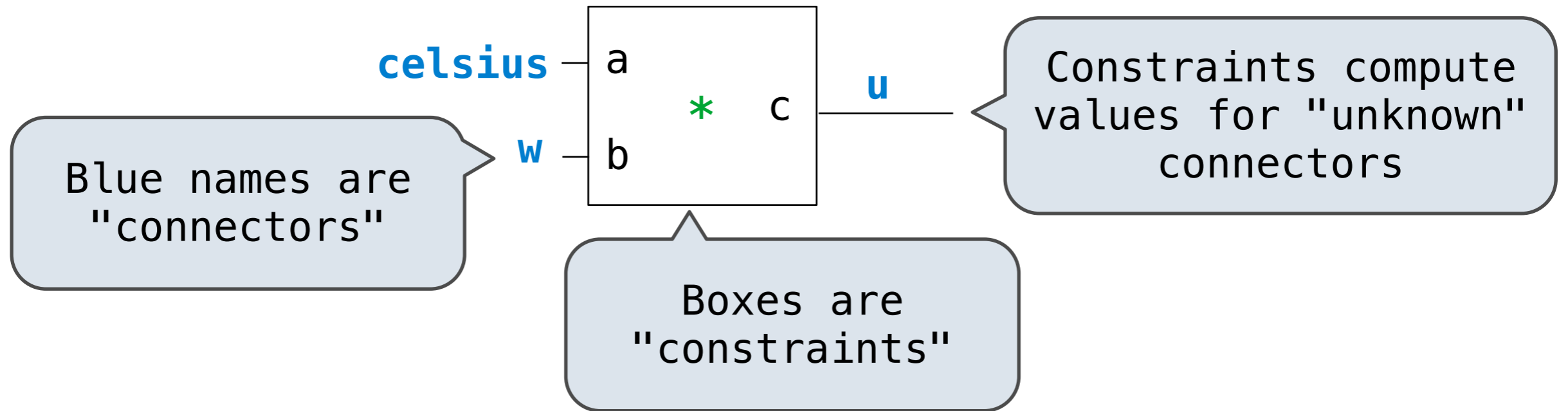
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- A constraint can receive two messages from its connectors:

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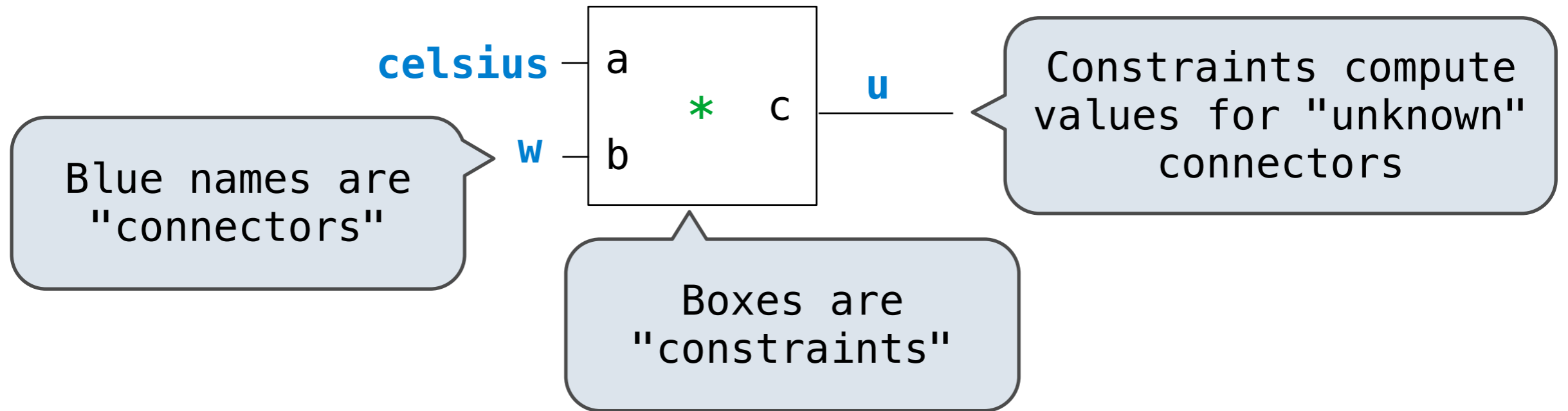
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  - **'new\_val'** indicates that some connector that is connected to the constraint has a new value.

# Anatomy of a Constraint

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- *Connectors* represent quantities that have values.
- *Constraints* spread information among connectors.
- A constraint can receive two messages from its connectors:
  - **'new\_val'** indicates that some connector that is connected to the constraint has a new value.
  - **'forget'** indicates that some connector that is connected to the constraint has forgotten its value.

# Constructing a Constraint Network

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```
celsius = make_connector('Celsius')
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# Constructing a Constraint Network

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**celsius**

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celsius = make_connector('Celsius')
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celsius = make_connector('Celsius')  
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celsius = make_connector('Celsius')  
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make_converter(celsius, fahrenheit)
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```
def make_converter(celsius, fahrenheit):  
    """Make a temperature conversion network."""
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# Constructing a Constraint Network

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```
def make_converter(celsius, fahrenheit):  
    """Make a temperature conversion network."""  
    u, v, w, x, y = [make_connector() for _ in range(5)]
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**u**

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    multiplier(celsius, w, u)
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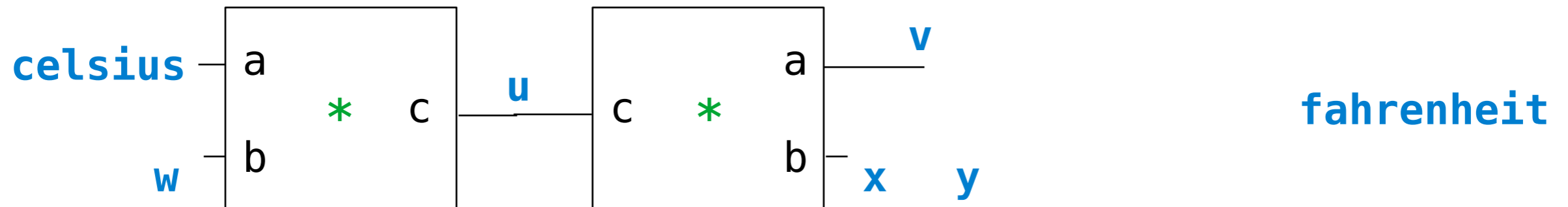


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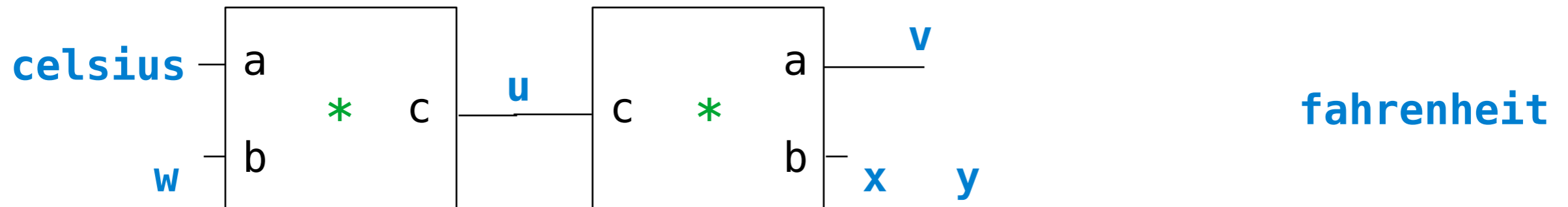


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    adder(v, y, fahrenheit)
```

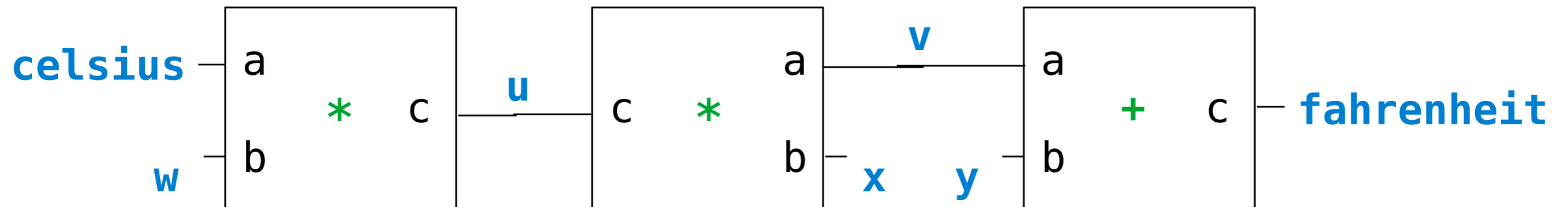


```
celsius = make_connector('Celsius')  
fahrenheit = make_connector('Fahrenheit')  
make_converter(celsius, fahrenheit)
```

# Constructing a Constraint Network

---

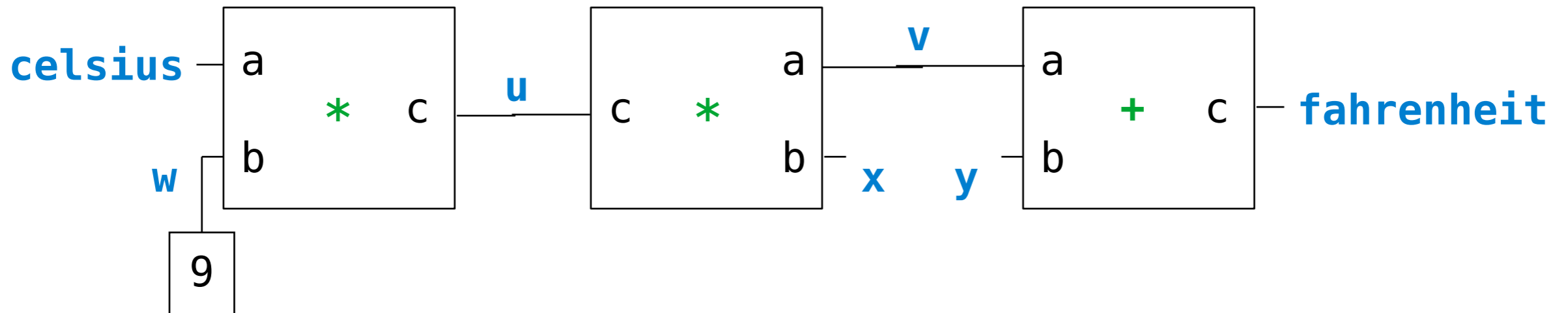
```
def make_converter(celsius, fahrenheit):  
    """Make a temperature conversion network."""  
    u, v, w, x, y = [make_connector() for _ in range(5)]  
    multiplier(celsius, w, u)  
    multiplier(v, x, u)  
    adder(v, y, fahrenheit)
```



```
celsius = make_connector('Celsius')  
fahrenheit = make_connector('Fahrenheit')  
make_converter(celsius, fahrenheit)
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# Constructing a Constraint Network

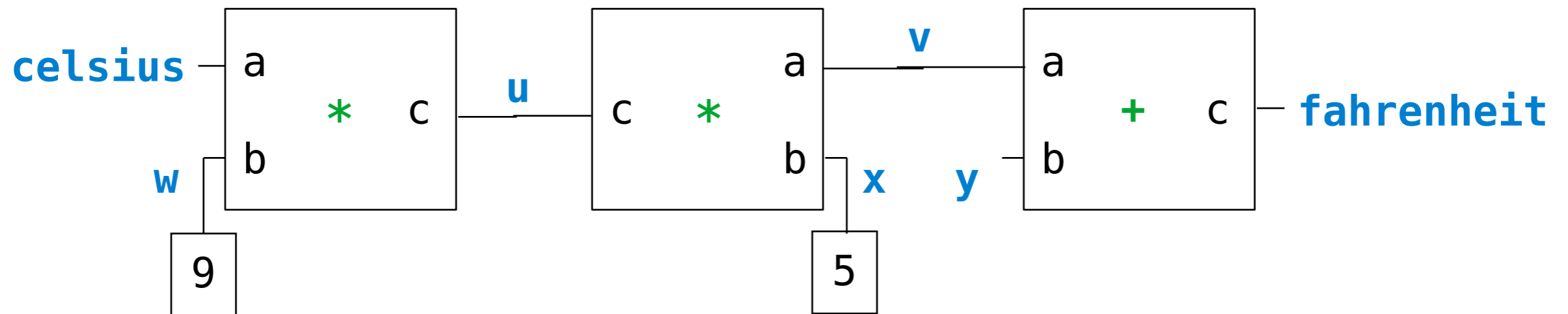
```
def make_converter(celsius, fahrenheit):  
    """Make a temperature conversion network."""  
    u, v, w, x, y = [make_connector() for _ in range(5)]  
    multiplier(celsius, w, u)  
    multiplier(v, x, u)  
    adder(v, y, fahrenheit)  
    constant(w, 9)
```



```
celsius = make_connector('Celsius')  
fahrenheit = make_connector('Fahrenheit')  
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# Constructing a Constraint Network

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    adder(v, y, fahrenheit)  
    constant(w, 9)  
    constant(x, 5)
```

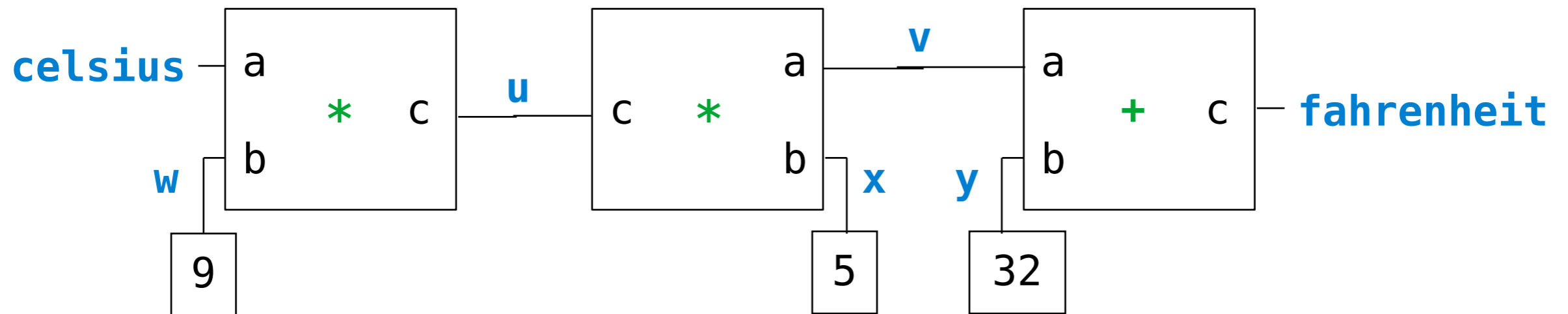


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# Constructing a Constraint Network

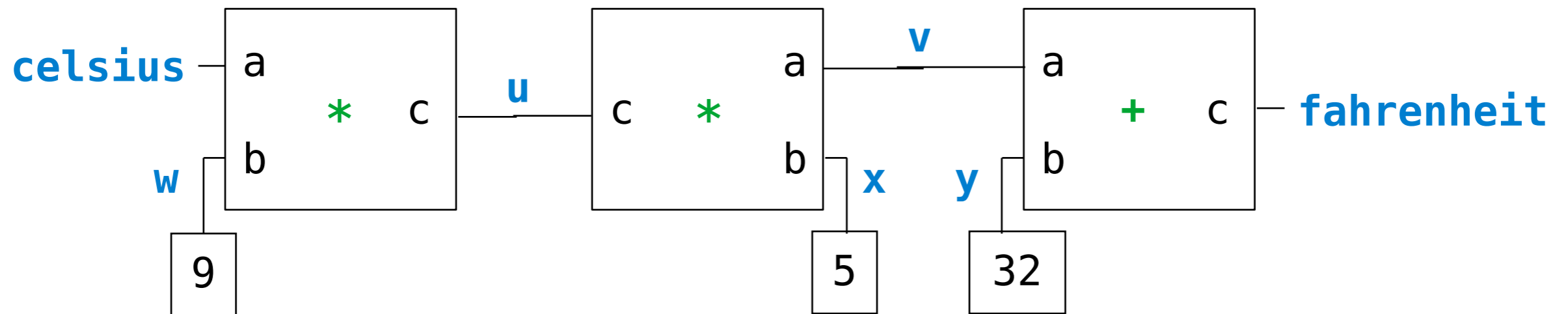
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    constant(w, 9)  
    constant(x, 5)  
    constant(y, 32)
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fahrenheit = make_connector('Fahrenheit')  
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```

Demo

# The Messages of a Connector

---

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```
connector = make_connector('Celsius')
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`connector['val']` is the current value of the connector.

`connector['forget'](source)` tells the connector that the source is requesting it to forget its value.

`connector['connect'](source)` tells the connector to participate in a new constraint, the source.

# Implementing an Adder Constraint

---

# Implementing an Adder Constraint

---

```
def adder_constraint(a, b, c):  
    """The constraint that  $a + b = c$ .  
  
    >>> a, b, c = [make_connector(name) for name in ('a', 'b', 'c')]  
    >>> constraint = adder_constraint(a, b, c)  
    >>> a['set_val']('user', 2)  
    a = 2  
    >>> b['set_val']('user', 3)  
    b = 3  
    c = 5  
    """
```

# Implementing an Adder Constraint

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def adder_constraint(a, b, c):  
    """The constraint that  $a + b = c$ .  
  
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    a = 2  
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    b = 3  
    c = 5  
    """  
  
def new_value():
```

# Implementing an Adder Constraint

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    a = 2  
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    b = 3  
    c = 5  
    """  
  
def new_value():  
    # We will implement this function momentarily!
```

# Implementing an Adder Constraint

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    >>> a, b, c = [make_connector(name) for name in ('a', 'b', 'c')]
    >>> constraint = adder_constraint(a, b, c)
    >>> a['set_val']('user', 2)
    a = 2
    >>> b['set_val']('user', 3)
    b = 3
    c = 5
    """

    def new_value():
        # We will implement this function momentarily!

    def forget_value():
        for connector in (a, b, c):
            connector['forget'](constraint)

    constraint = {'new_val': new_value, 'forget': forget_value}

    for connector in (a, b, c):
        connector['connect'](constraint)

    return constraint
```

# Generalizing to a Multiplication Constraint

---

# Generalizing to a Multiplication Constraint

---

```
from operator import add, sub, mul, truediv

def adder(a, b, c):
    """The constraint that  $a + b = c$ ."""
    return make_ternary_constraint(a, b, c, add, sub, sub)

def multiplier(a, b, c):
    """The constraint that  $a * b = c$ ."""
    return make_ternary_constraint(a, b, c, mul, truediv, truediv)
```



# Generalizing to a Multiplication Constraint

---

```
def make_ternary_constraint(a, b, c, ab, ca, cb):  
    """The constraint that  $ab(a,b)=c$  and  $ca(c,a)=b$  and  $cb(c,b)=a$ ."""  
    def new_value():  
        av, bv, cv = [connector['has_val']() for connector in (a, b, c)]
```

```
from operator import add, sub, mul, truediv
```

```
def adder(a, b, c):  
    """The constraint that  $a + b = c$ ."""  
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# Generalizing to a Multiplication Constraint

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## Connectors

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def make_ternary_constraint(a, b, c, ab, ca, cb):  
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```
from operator import add, sub, mul, truediv
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# Generalizing to a Multiplication Constraint

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Connectors

Relations

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```
from operator import add, sub, mul, truediv
```

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def adder(a, b, c):  
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# Generalizing to a Multiplication Constraint

---

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    """The constraint that  $ab(a,b)=c$  and  $ca(c,a)=b$  and  $cb(c,b)=a$ ."""  
    def new_value():  
        av, bv, cv = [connector['has_val']() for connector in (a, b, c)]  
        if av and bv:  
            c['set_val'](constraint, ab(a['val'], b['val']))
```

```
from operator import add, sub, mul, truediv
```

```
def adder(a, b, c):  
    """The constraint that  $a + b = c$ ."""  
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# Generalizing to a Multiplication Constraint

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    """The constraint that  $ab(a,b)=c$  and  $ca(c,a)=b$  and  $cb(c,b)=a$ ."""
    def new_value():
        av, bv, cv = [connector['has_val']() for connector in (a, b, c)]

        if av and bv:
            c['set_val'](constraint, ab(a['val'], b['val']))

        elif av and cv:
            b['set_val'](constraint, ca(c['val'], a['val']))

    return constraint

from operator import add, sub, mul, truediv

def adder(a, b, c):
    """The constraint that  $a + b = c$ ."""
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# Generalizing to a Multiplication Constraint

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Connectors

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    def new_value():
        av, bv, cv = [connector['has_val']() for connector in (a, b, c)]

        if av and bv:
            c['set_val'](constraint, ab(a['val'], b['val']))

        elif av and cv:
            b['set_val'](constraint, ca(c['val'], a['val']))

        elif bv and cv:
            a['set_val'](constraint, cb(c['val'], b['val']))

    return constraint

from operator import add, sub, mul, truediv

def adder(a, b, c):
    """The constraint that  $a + b = c$ ."""
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```

# Implementing a Connector

---

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---

```
def make_connector(name=None):
```



# Implementing a Connector

---

```
def make_connector(name=None):
```

```
connector = {'val': None,  
            'set_val': set_value,  
            'forget': forget_value,  
            'has_val': lambda: connector['val'] is not None,  
            'connect': lambda source: constraints.append(source)}
```

# Implementing a Connector

---

```
def make_connector(name=None):
```

```
    connector = {'val': None,  
                'set_val': set_value,  
                'forget': forget_value,  
                'has_val': lambda: connector['val'] is not None,  
                'connect': lambda source: constraints.append(source)}  
  
    return connector
```

# Implementing a Connector

---

```
def make_connector(name=None):
```

```
    informant = None
    constraints = []
```

```
    connector = {'val': None,
                 'set_val': set_value,
                 'forget': forget_value,
                 'has_val': lambda: connector['val'] is not None,
                 'connect': lambda source: constraints.append(source)}
```

```
    return connector
```

# Implementing a Connector

---

```
def make_connector(name=None):  
    informant = None  
    constraints = []  
  
    def set_value(source, value):  
        nonlocal informant  
        val = connector['val']  
        if val is None:  
            informant, connector['val'] = source, value  
            if name is not None:  
                print(name, '=', value)  
            inform_all_except(source, 'new_val', constraints)  
        else:  
            if val != value:  
                print('Contradiction detected:', val, 'vs', value)  
  
connector = {'val': None,  
            'set_val': set_value,  
            'forget': forget_value,  
            'has_val': lambda: connector['val'] is not None,  
            'connect': lambda source: constraints.append(source)}  
  
return connector
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    informant = None  
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        if val is None:  
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            if name is not None:  
                print(name, '=', value)  
            inform_all_except(source, 'new_val', constraints)  
        else:  
            if val != value:  
                print('Contradiction detected:', val, 'vs', value)  
  
    def forget_value(source):  
        nonlocal informant  
        if informant == source:  
            informant, connector['val'] = None, None  
            if name is not None:  
                print(name, 'is forgotten')  
            inform_all_except(source, 'forget', constraints)  
  
    connector = {'val': None,  
                'set_val': set_value,  
                'forget': forget_value,  
                'has_val': lambda: connector['val'] is not None,  
                'connect': lambda source: constraints.append(source)}  
  
    return connector
```