61A Lecture 21

Monday, October 17

Space Consumption

Which environment frames do we need to keep during evaluation?

Each step of evaluation has a set of **active** environments.

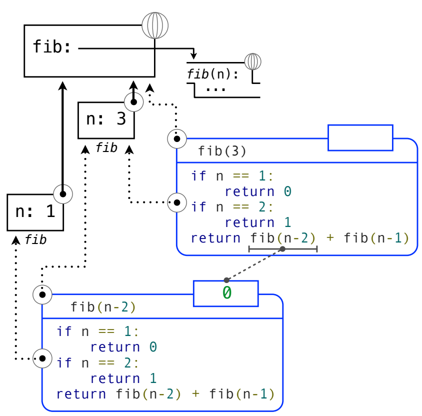
Values and frames referenced by active environments are kept.

Memory used for other values and frames can be reclaimed.

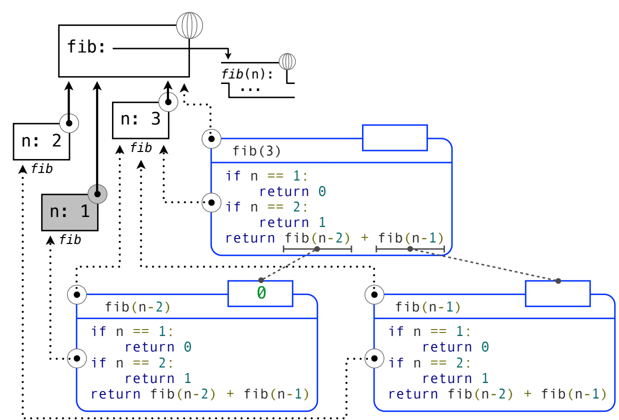
Active environments:

- The environment for the current expression being evaluated
- Environments for calls that depend upon the value of the current expression
- Environments associated with functions referenced by active environments

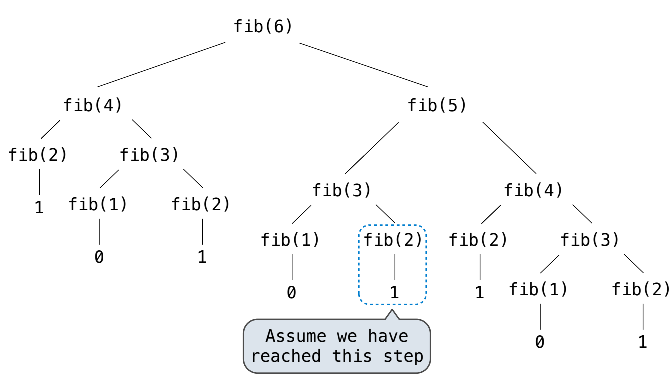
Fibonacci Environment Diagram



Fibonacci Environment Diagram

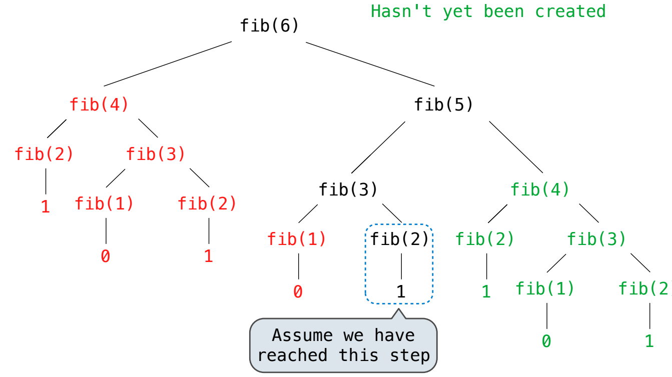


Fibonacci Memory Consumption

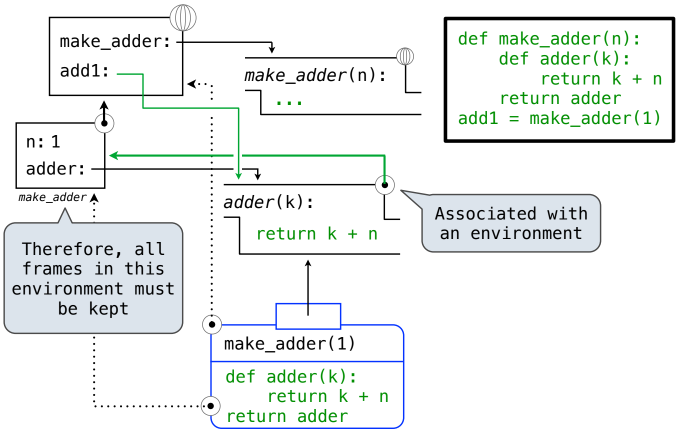


Fibonacci Memory Consumption

Has an active environment
Can be reclaimed
Hasn't yet been created



Active Environments for Returned Functions



Order of Growth

A method for bounding the resources used by a function as the "size" of a problem increases

n**:** size of the problem

***R*(*n*):** Measurement of some resource used (time or space)

$$R(n) = \Theta(f(n))$$

means that there are constants k_1 and k_2 such that

$$k_1 \cdot f(n) \leq R(n) \leq k_2 \cdot f(n)$$

for sufficiently large values of ***n*****.**

Iteration vs Memoized Tree Recursion

Iterative and memoized implementations are not the same.

	Time	**Space**
`fib_iter`	$\Theta(n)$	$\Theta(1)$
`fib` (memoized)	$\Theta(n)$	$\Theta(n)$

```
def fib_iter(n):
    prev, curr = 1, 0
    for _ in range(n-1):
         prev, curr = curr, prev + curr
    return curr
```

```
@memo
def fib(n):
    if n == 1:
        return 0
    if n == 2:
        return 1
    return fib(n-2) + fib(n-1)
```


Comparing orders of growth

$\Theta(b^n)$ Exponential growth! Recursive fib takes

$\Theta(\phi^n)$ steps, where $\phi = \frac{1+\sqrt{5}}{2} \approx 1.61828$

Incrementing the problem scales R(n) by a factor.

$\Theta(n)$ Linear growth. Resources scale with the problem.

$\Theta(\log n)$ Logarithmic growth. These functions scale well.

Doubling the problem increments resources needed.

$\Theta(1)$ Constant. The problem size doesn't matter.

Exponentiation

Goal: one more multiplication lets us double the problem size.

```
def exp(b, n):
    if n == 0:
        return 1
    return b * exp(b, n-1)
```

$$b^n = \begin{cases} 1 & \text{if } n = 0 \\ b \cdot b^{n-1} & \text{otherwise} \end{cases}$$

```
def square(x):
    return x*x

def fast_exp(b, n):
    if n == 0:
        return 1
    if n % 2 == 0:
        return square(fast_exp(b, n//2))
    else:
        return b * fast_exp(b, n-1)
```

$$b^n = \begin{cases} 1 & \text{if } n = 0 \\ (b^{\frac{1}{2}n})^2 & \text{if } n \text{ is even} \\ b \cdot b^{n-1} & \text{if } n \text{ is odd} \end{cases}$$

Exponentiation

Goal: one more multiplication lets us double the problem size.

	Time	**Space**
`exp`	$\Theta(n)$	$\Theta(n)$
`fast_exp`	$\Theta(\log n)$	$\Theta(\log n)$

```
def exp(b, n):
    if n == 0:
        return 1
    return b * exp(b, n-1)
```

```
def square(x):
    return x*x

def fast_exp(b, n):
    if n == 0:
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