# 61A Lecture 23

Friday, October 21

# Sets

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`One more built-in Python container type`

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{1, 2, 3, 4}
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{1, 2, 3, 4, 5}
>>> s.intersection({6, 5, 4, 3})
{3, 4}
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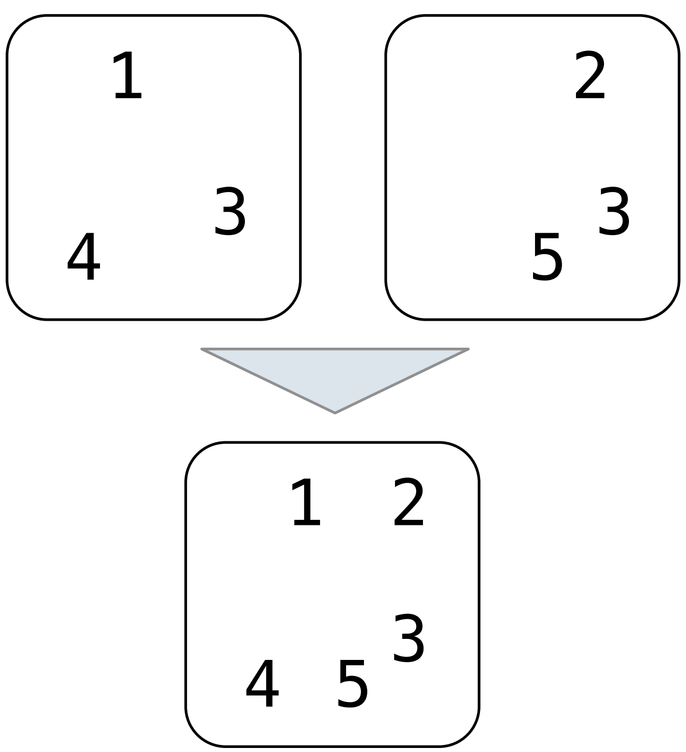

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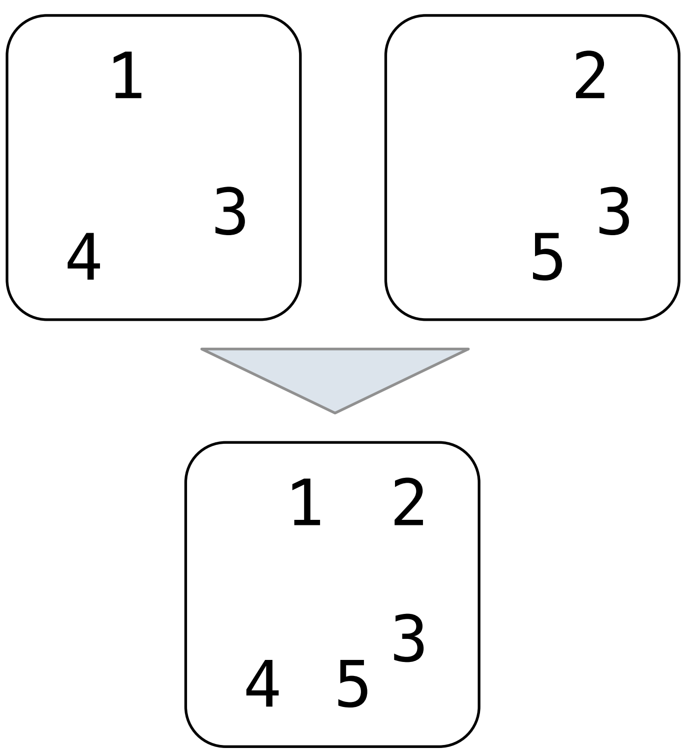

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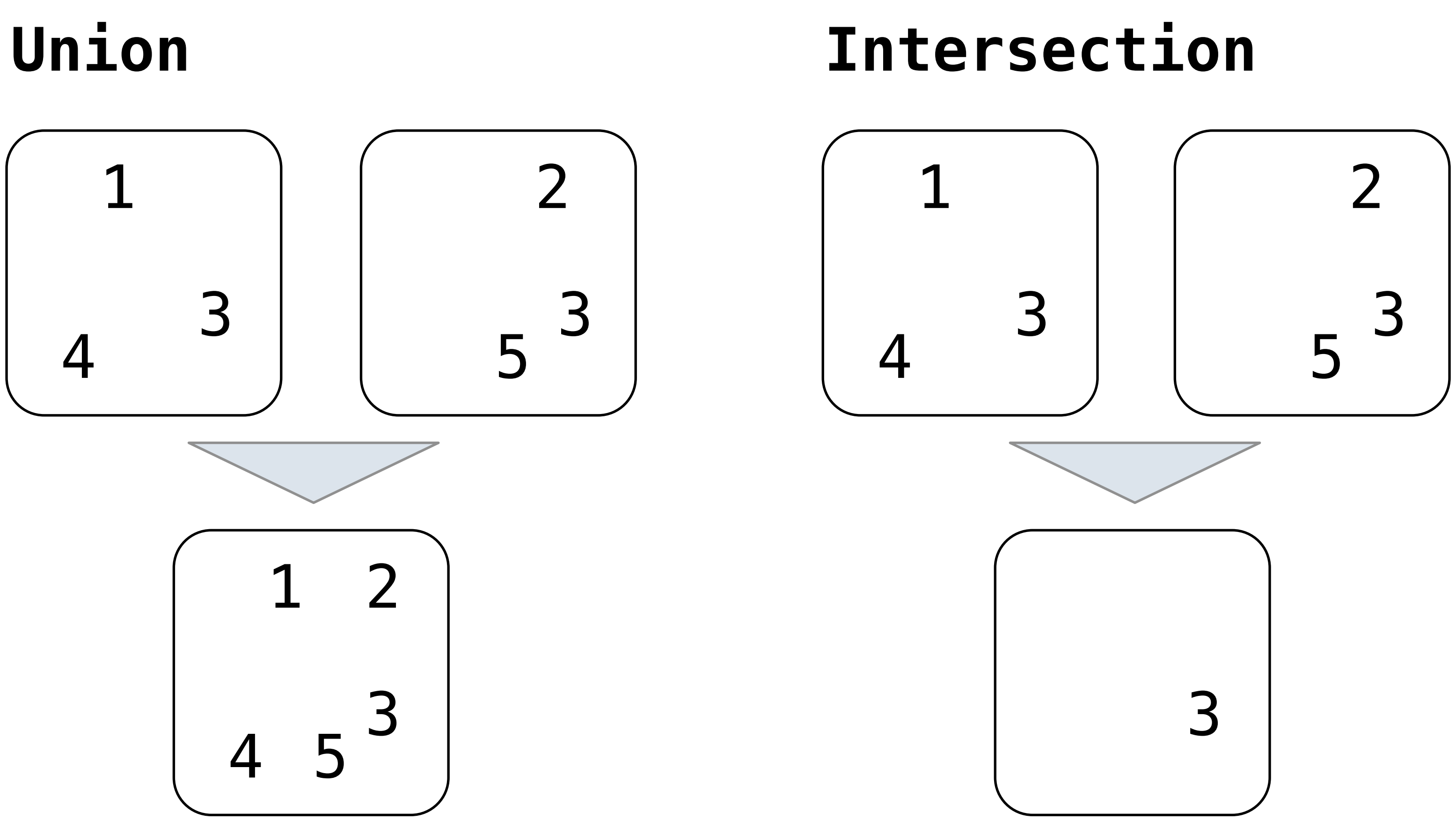

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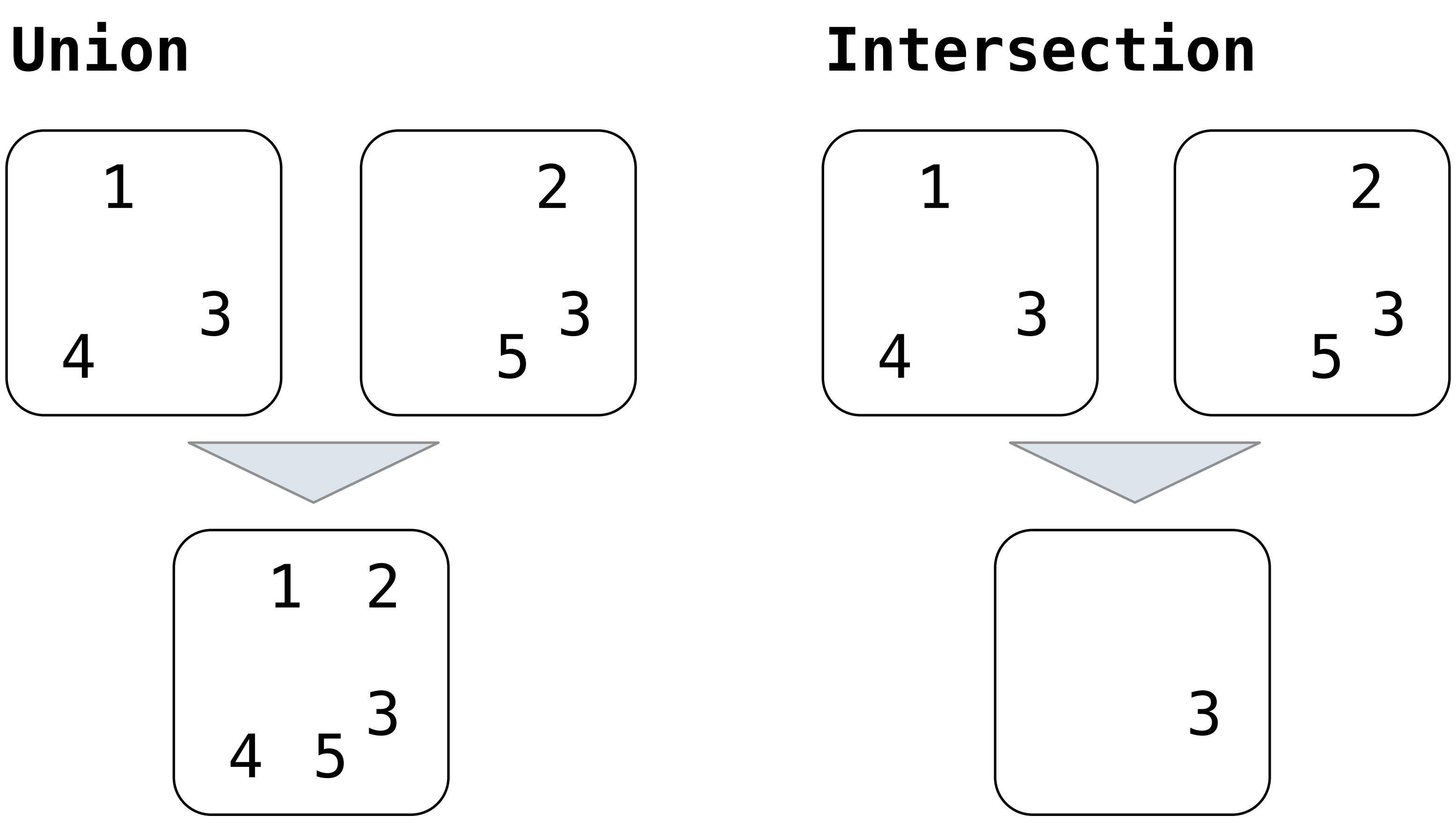

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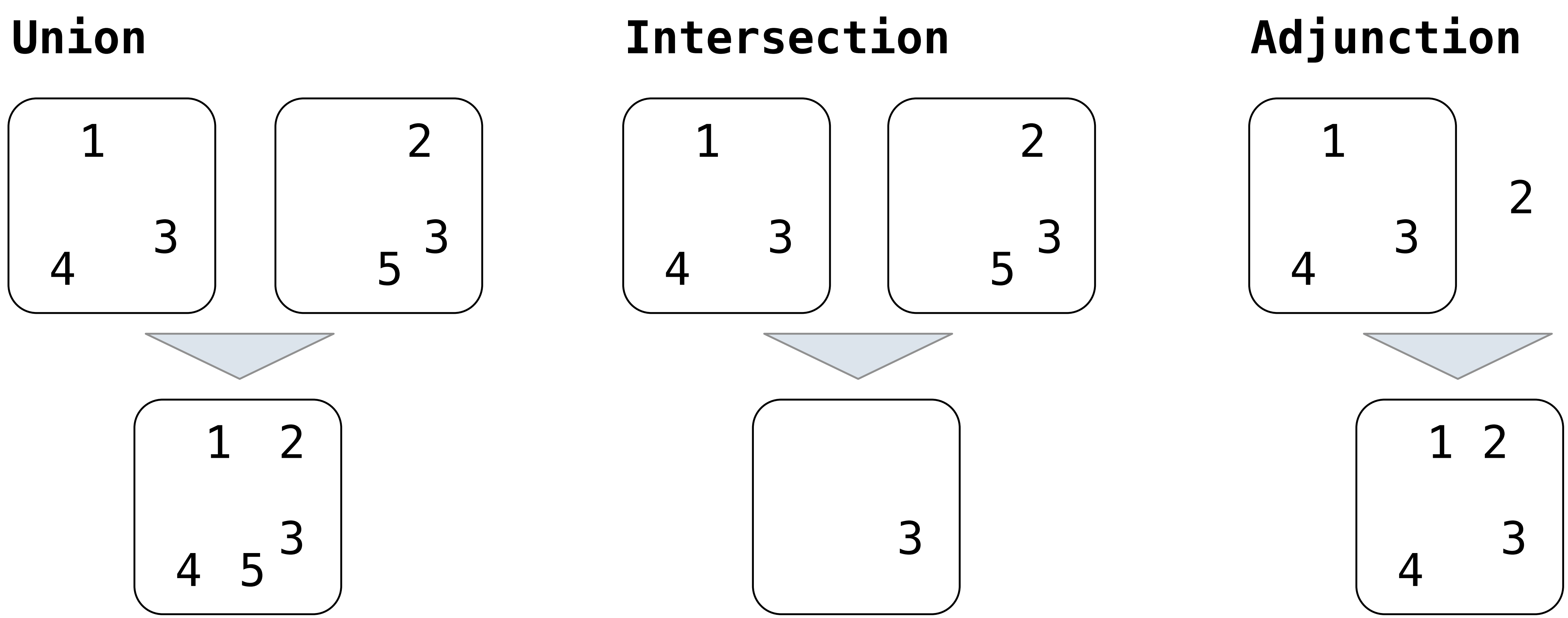

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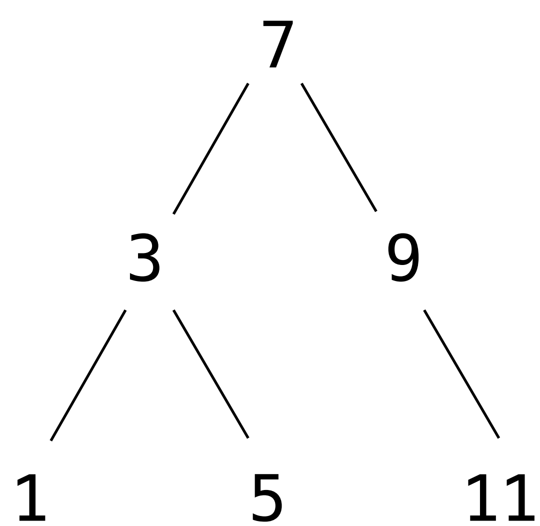

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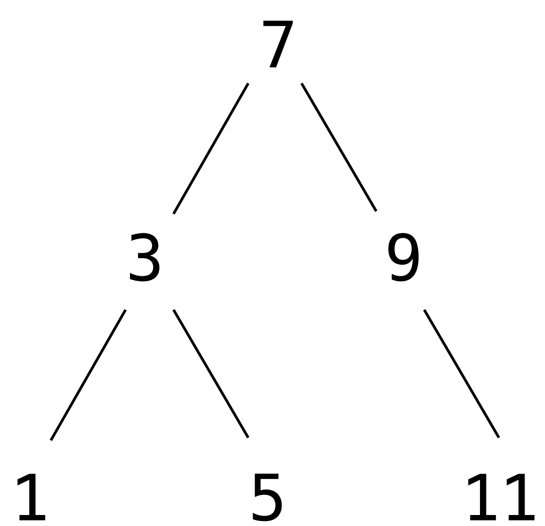


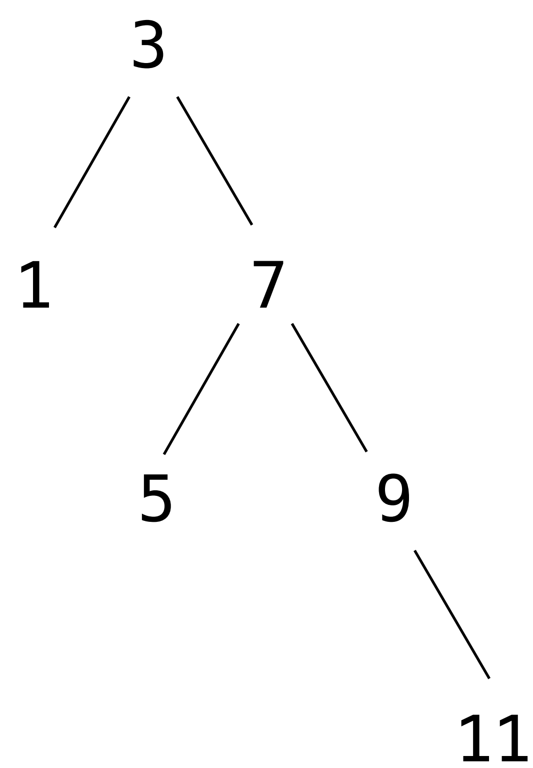

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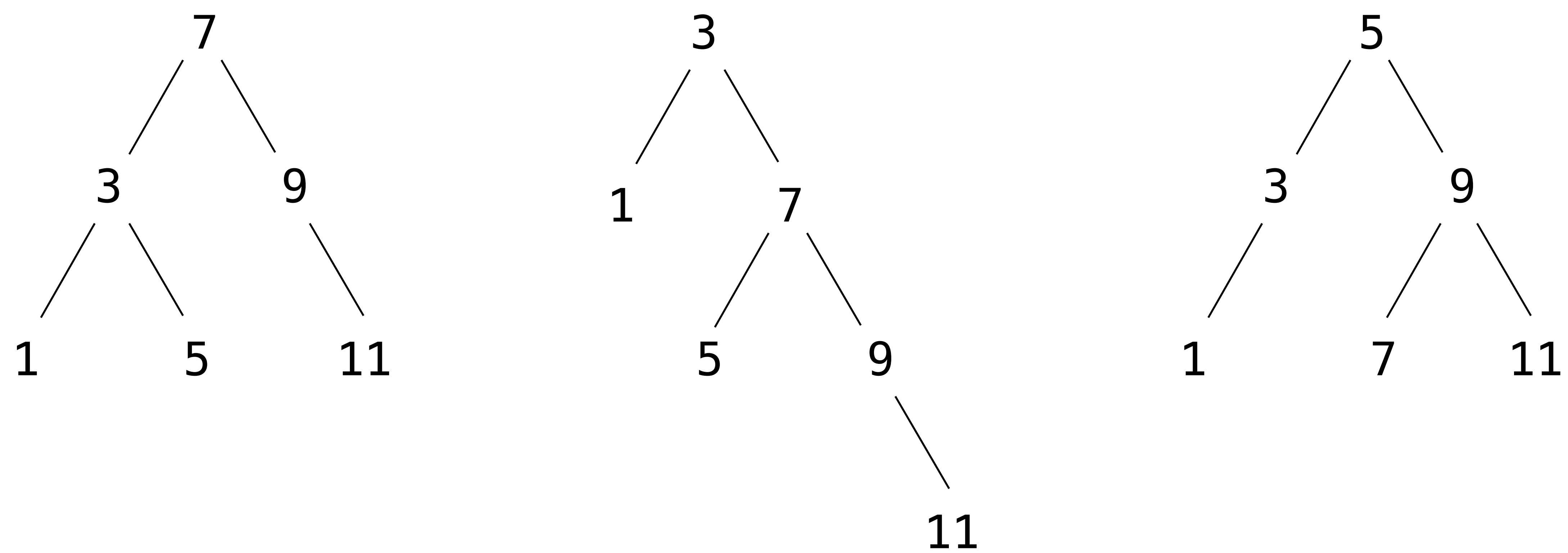

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def set_contains3(s, v):
    if s is None:
        return False
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```
def set_contains3(s, v):
    if s is None:
        return False
    elif s.entry == v:
        return True
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# Membership in Tree Sets

Set membership tests traverse the tree

- The element is either in the left or right sub-branch
- By focusing on one branch, we reduce the set by about half

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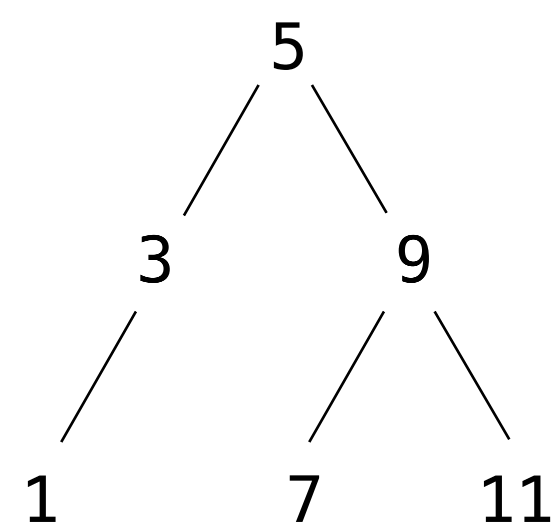

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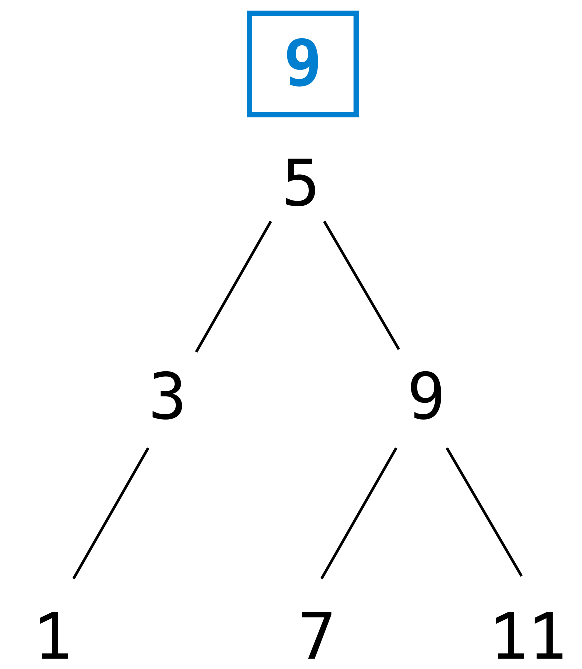

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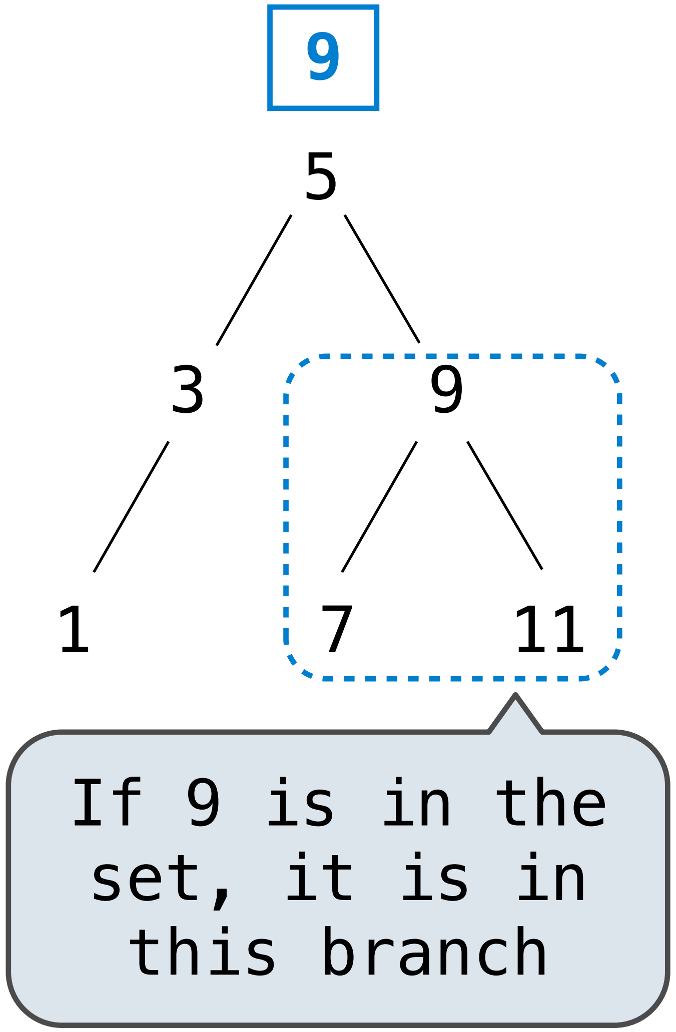

# Adjoining to a Tree Set

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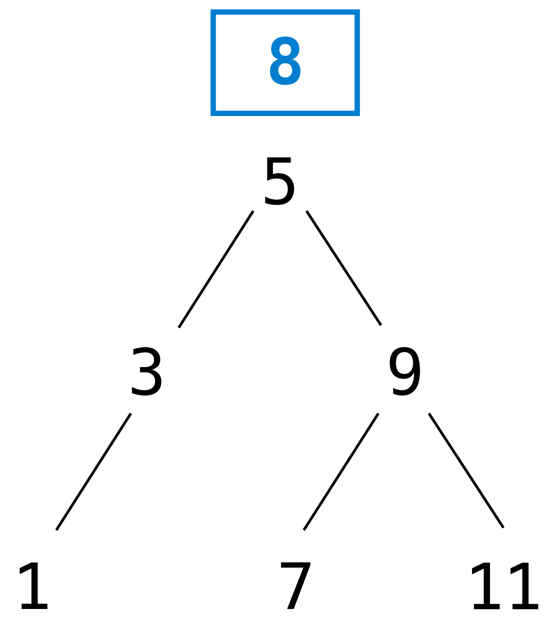




Friday, October 21, 2011

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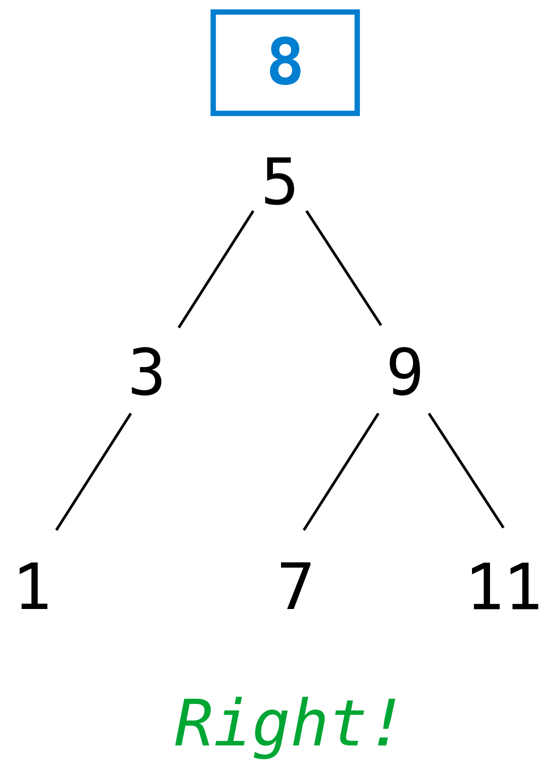

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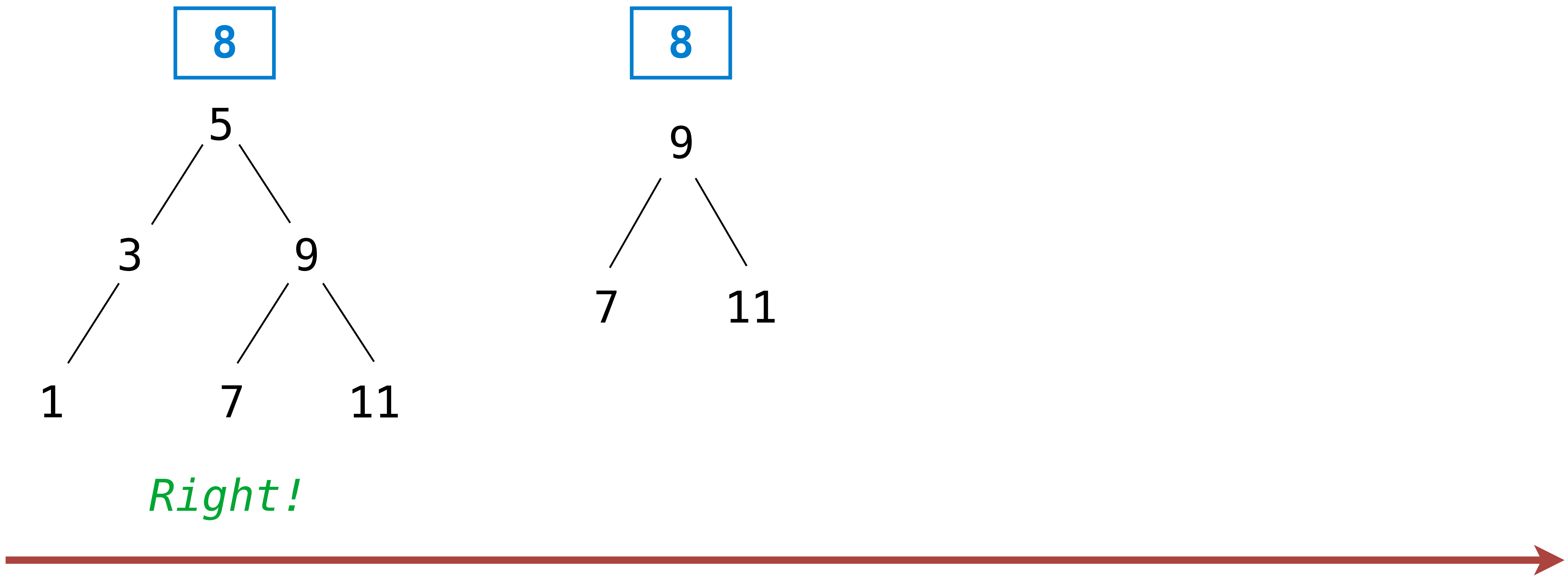

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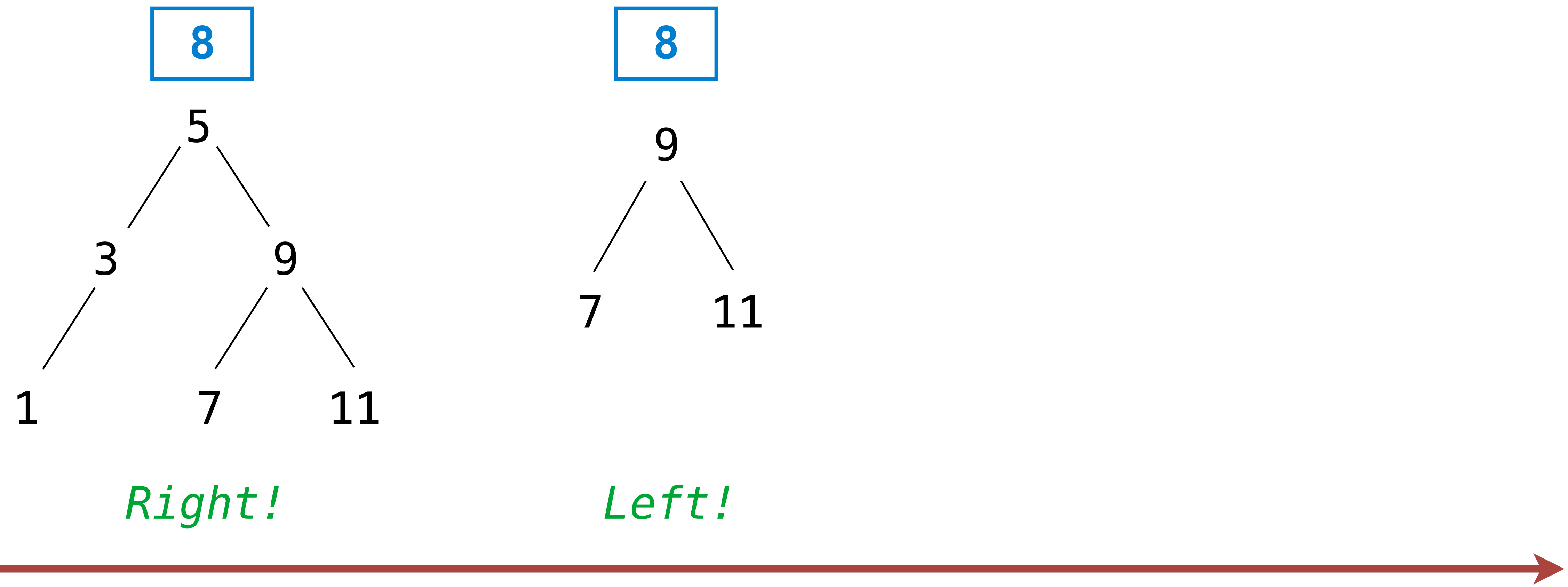

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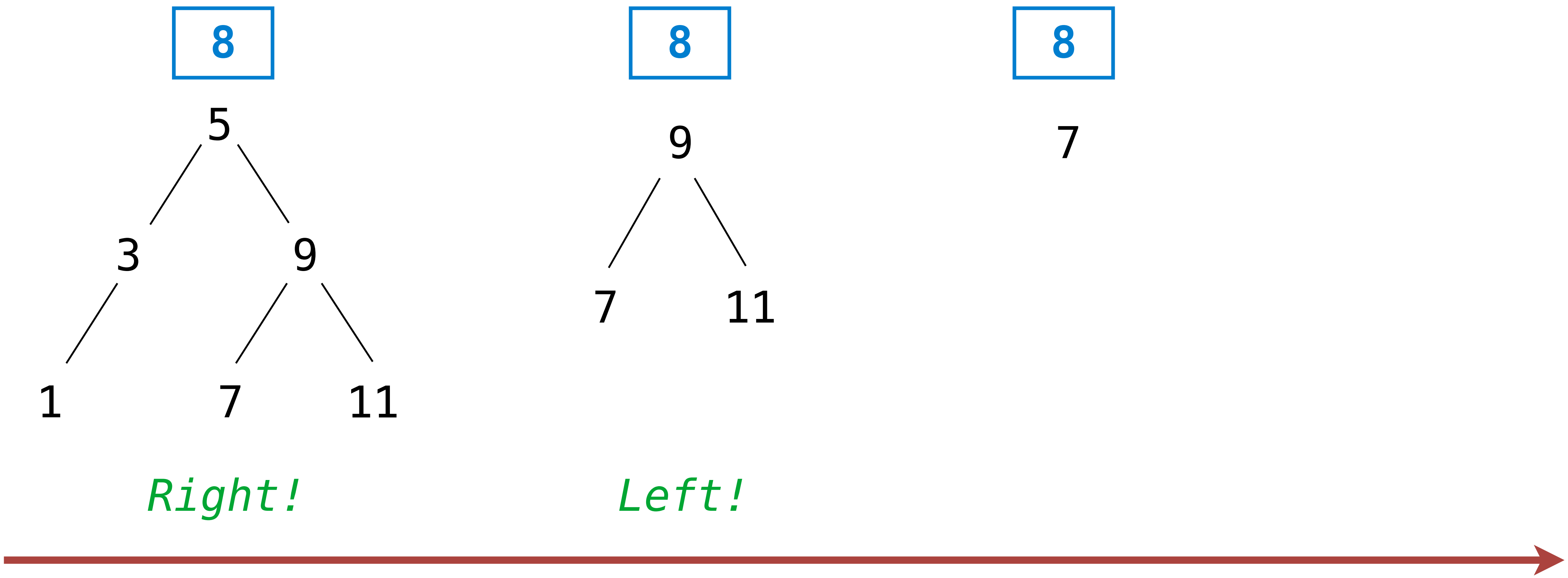

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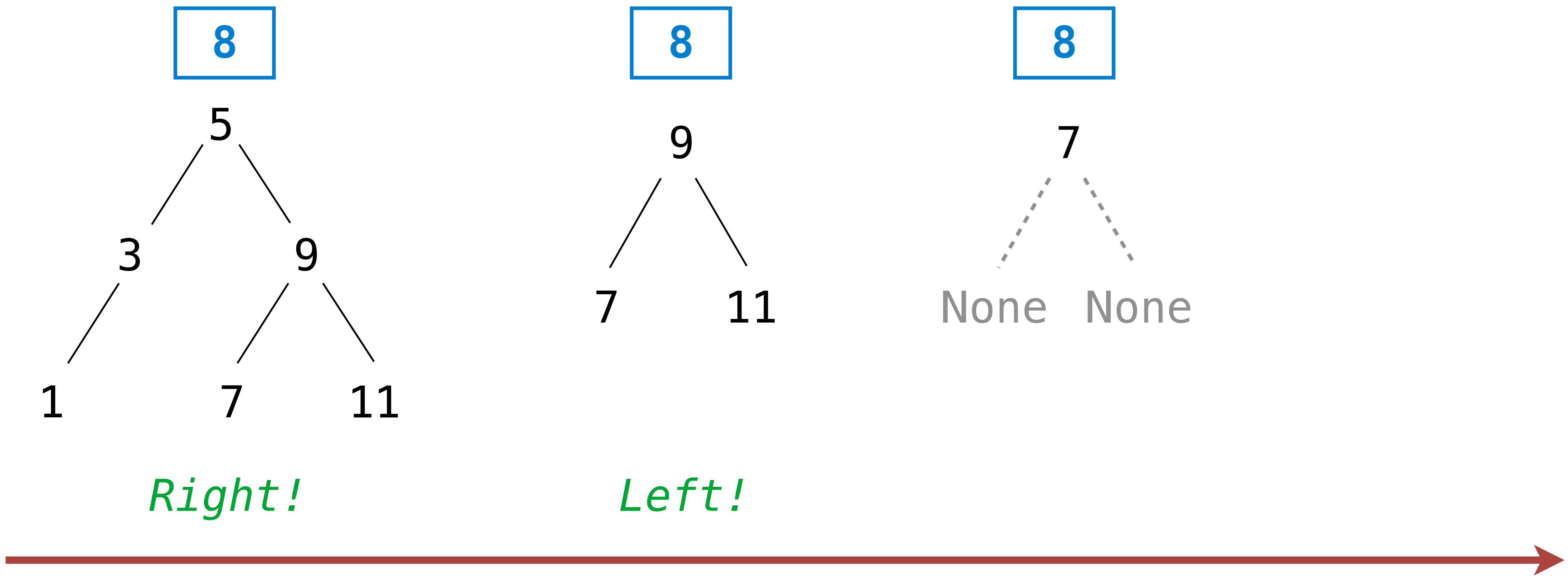

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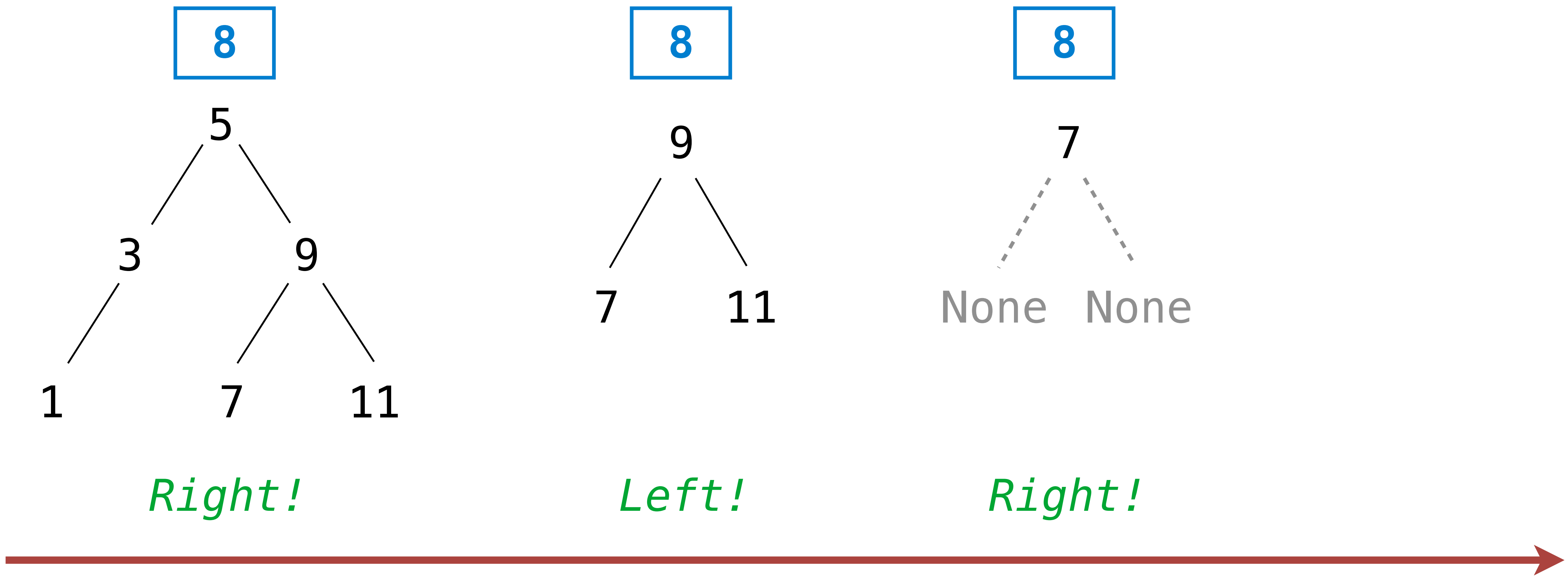

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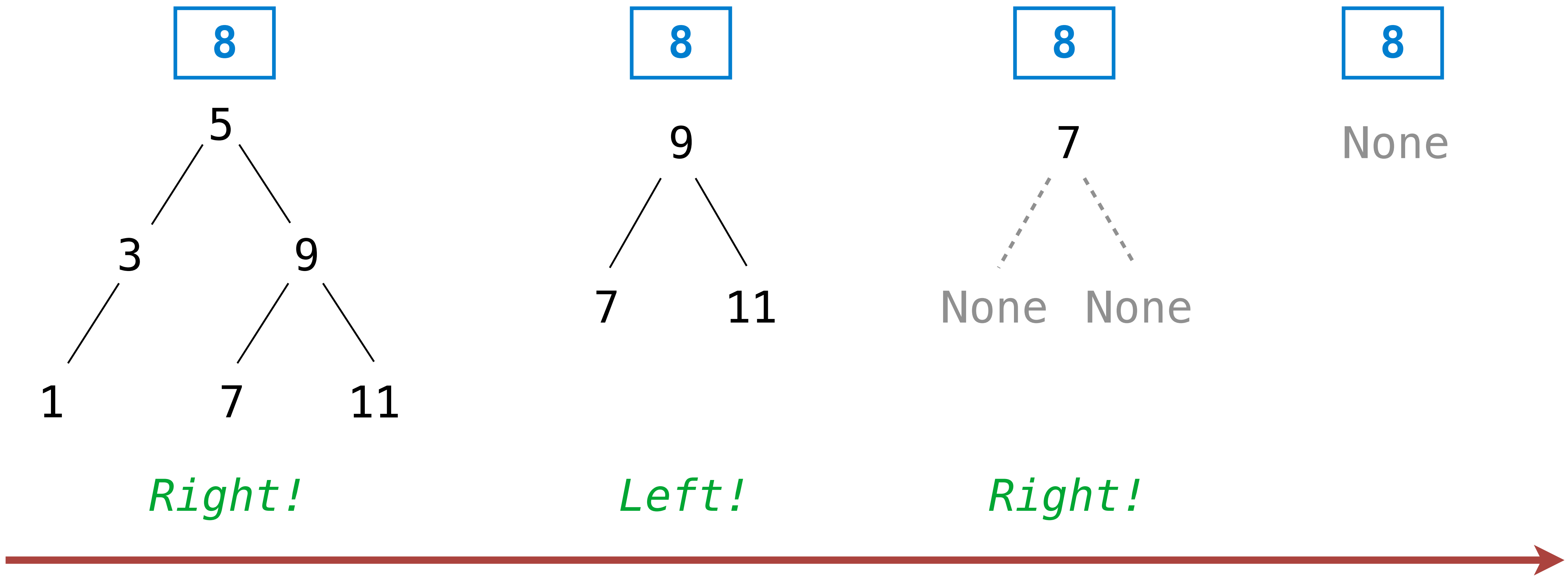

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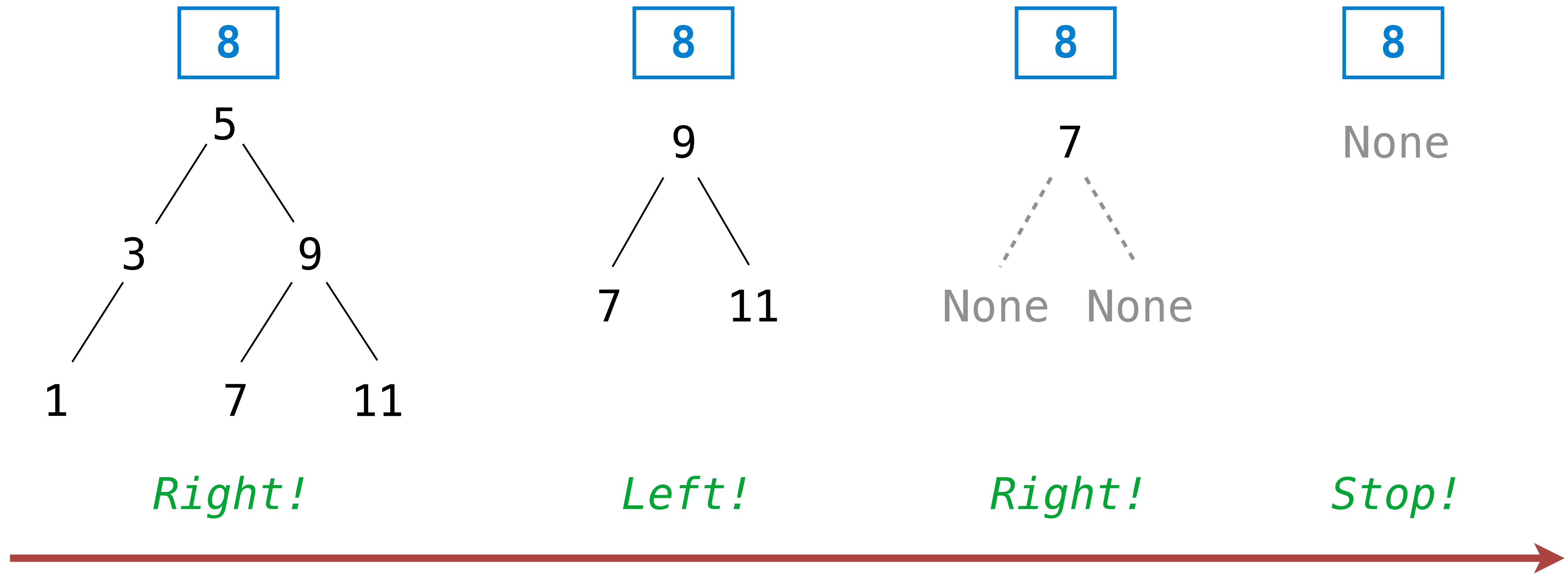

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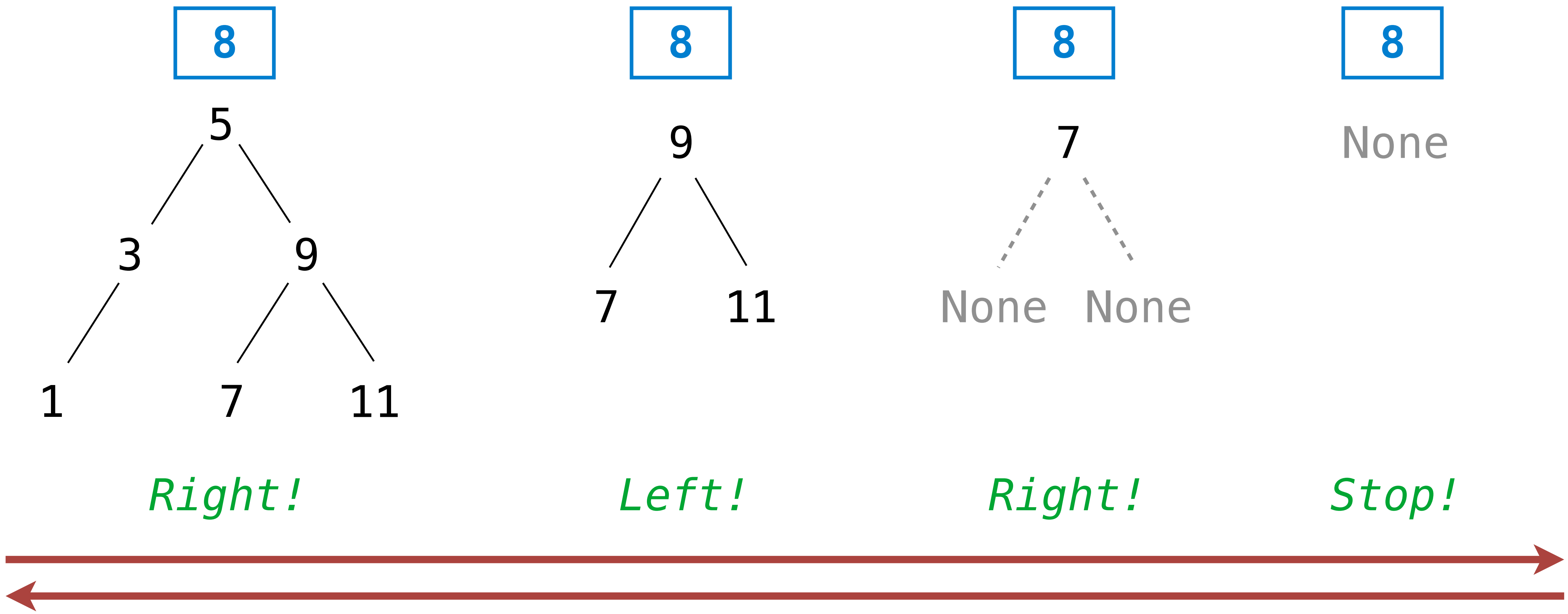

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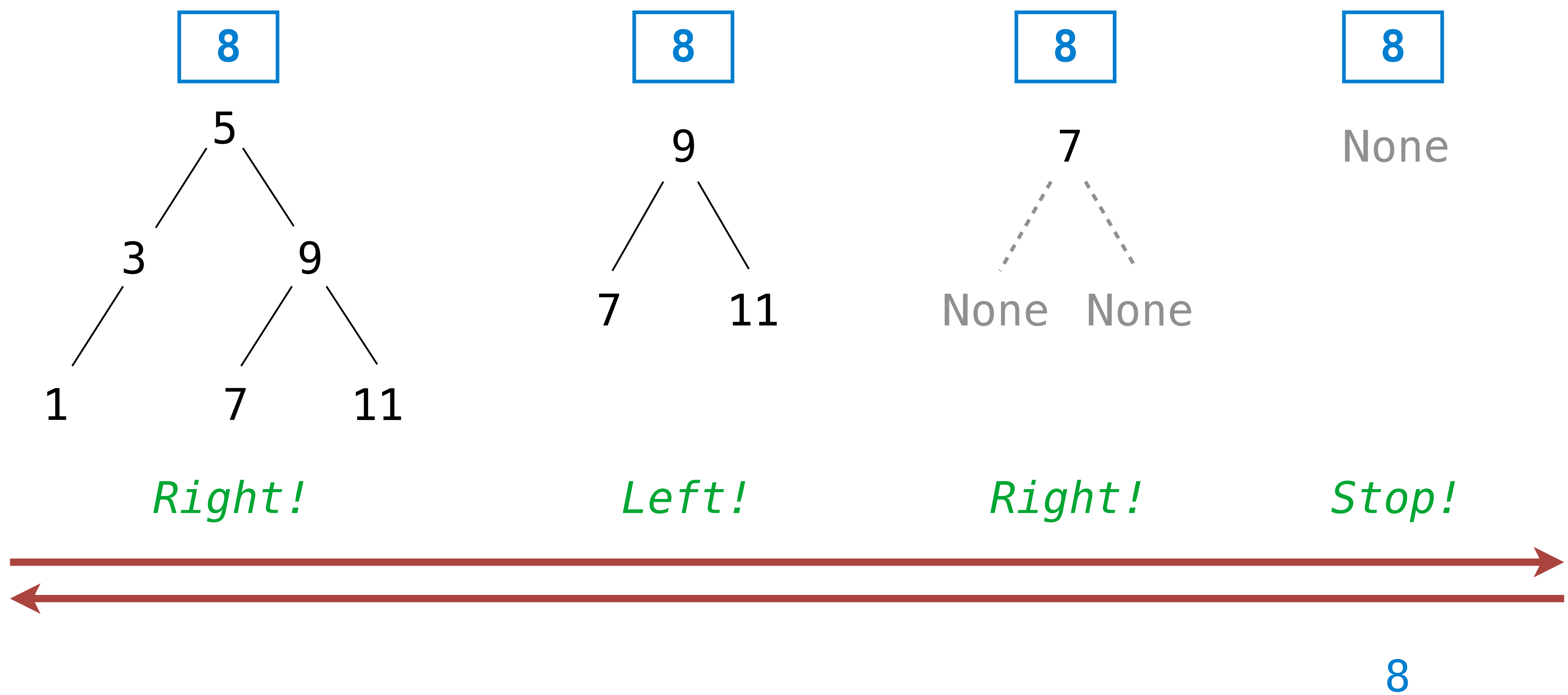

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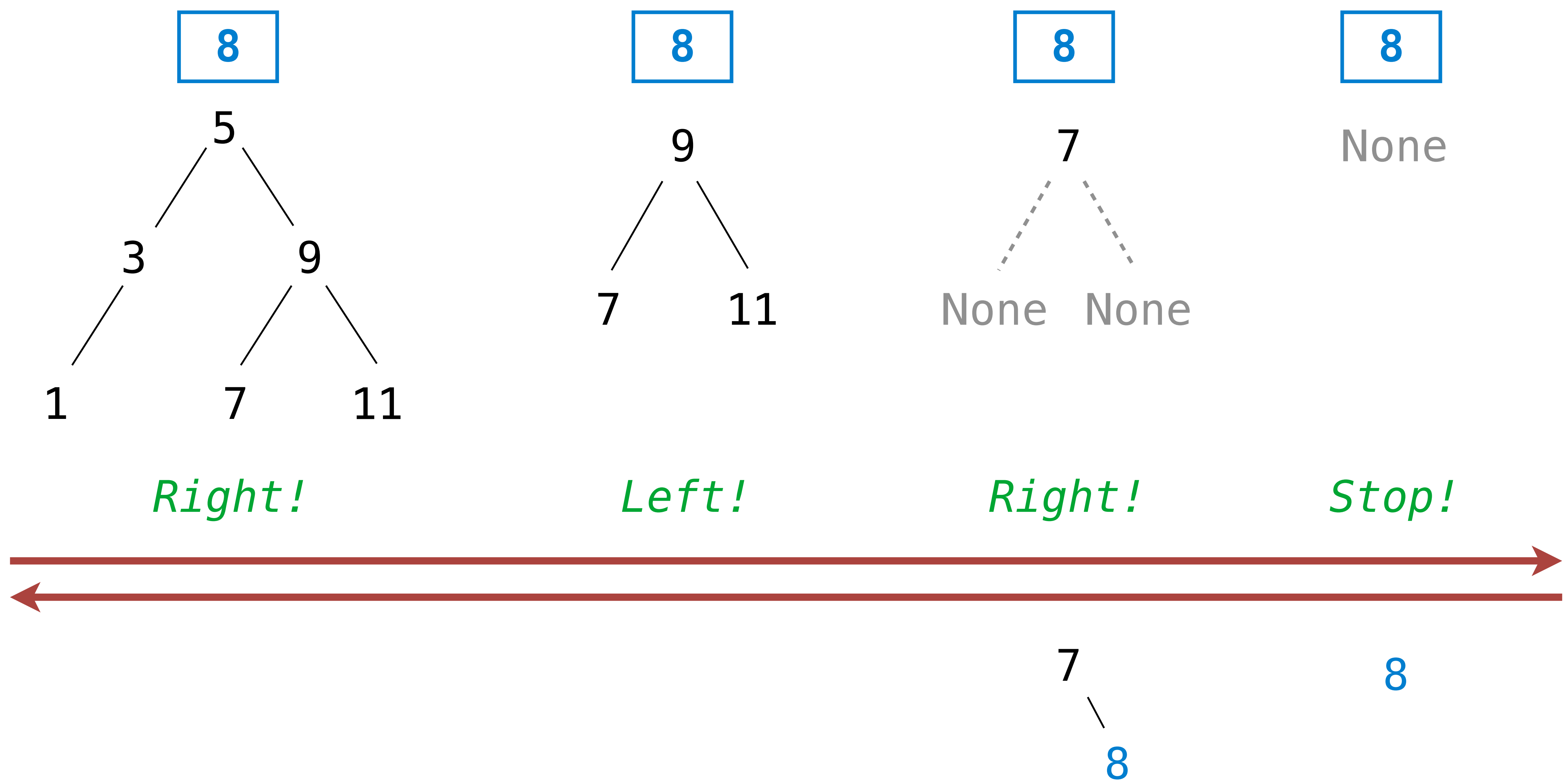

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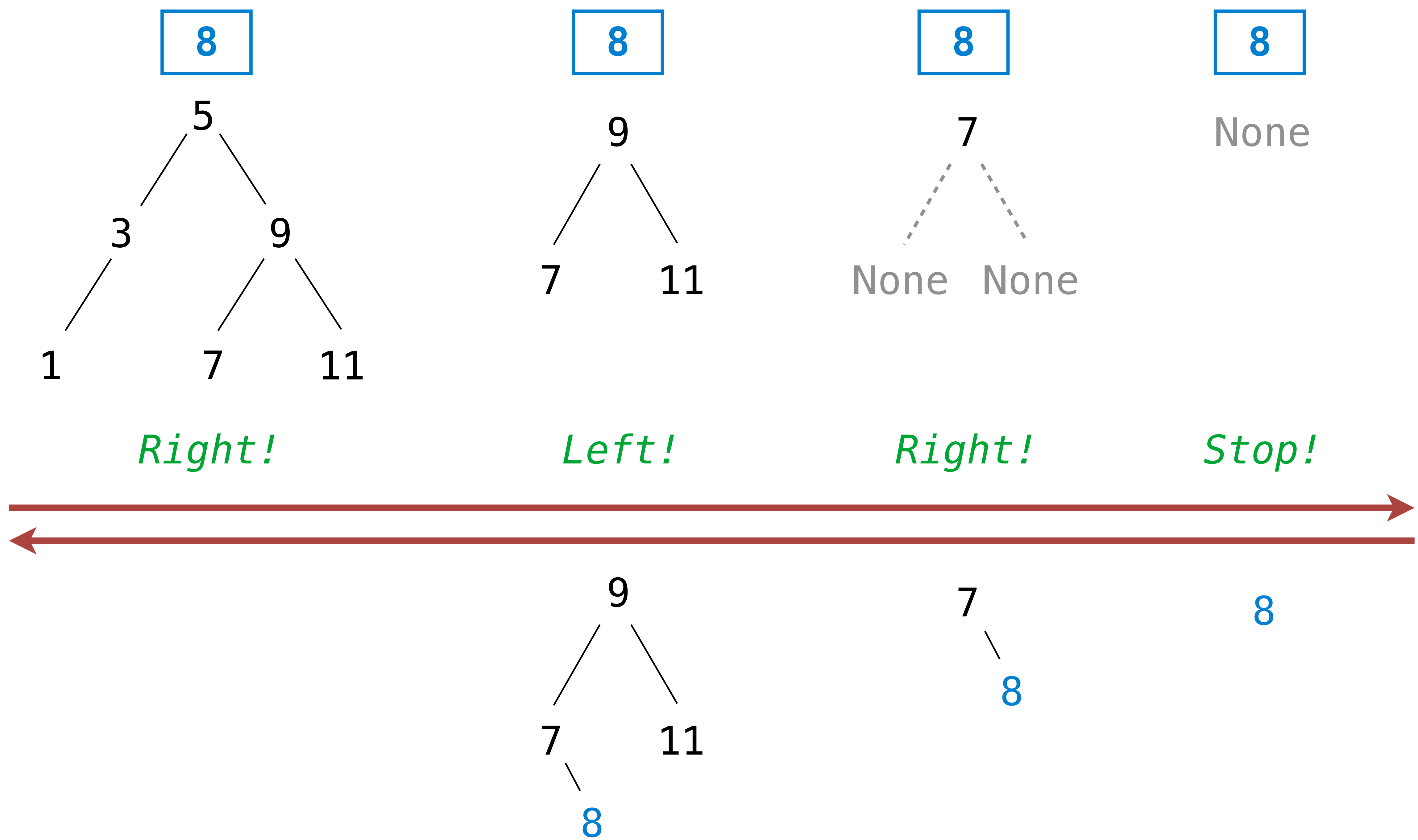

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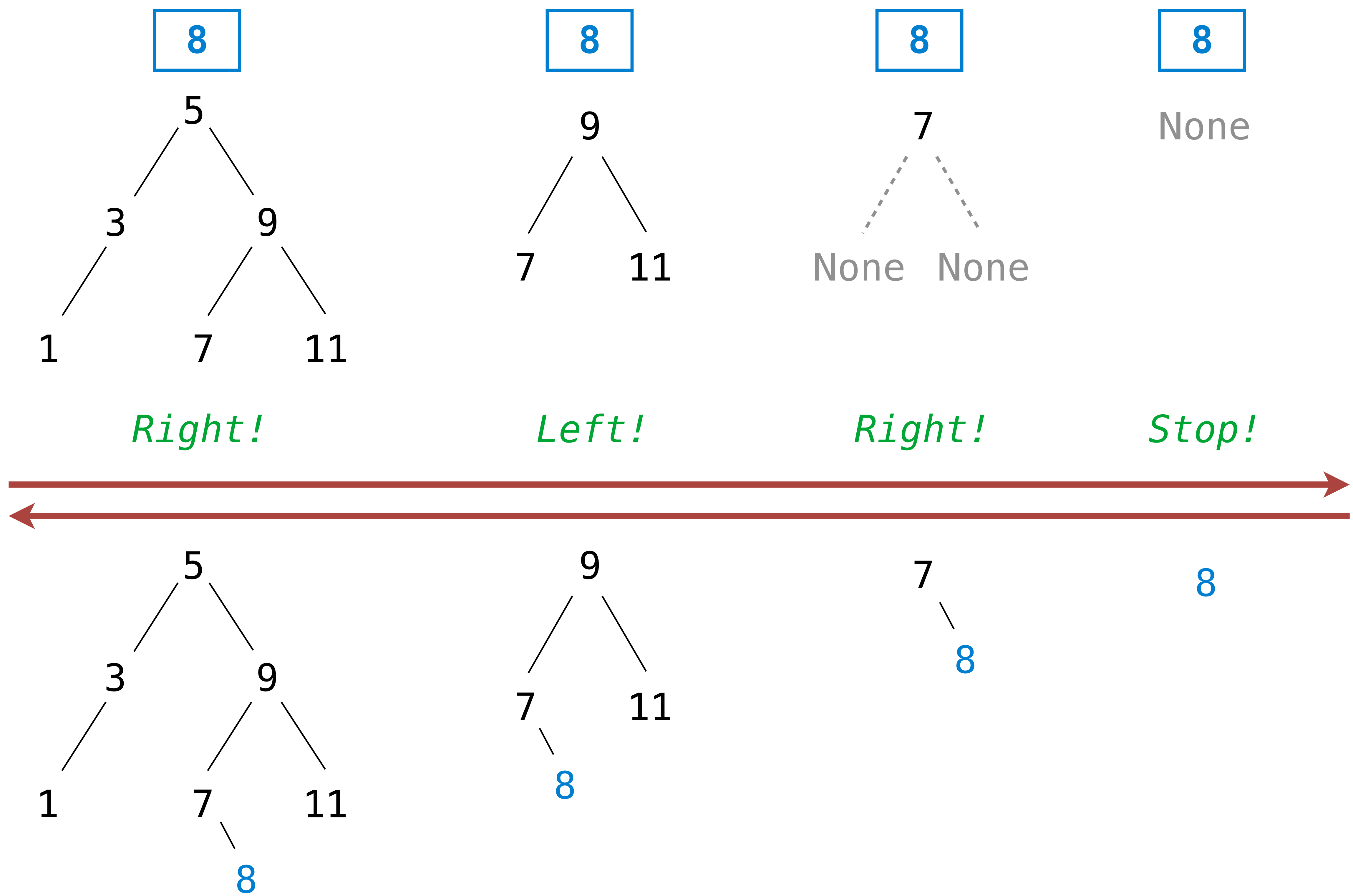

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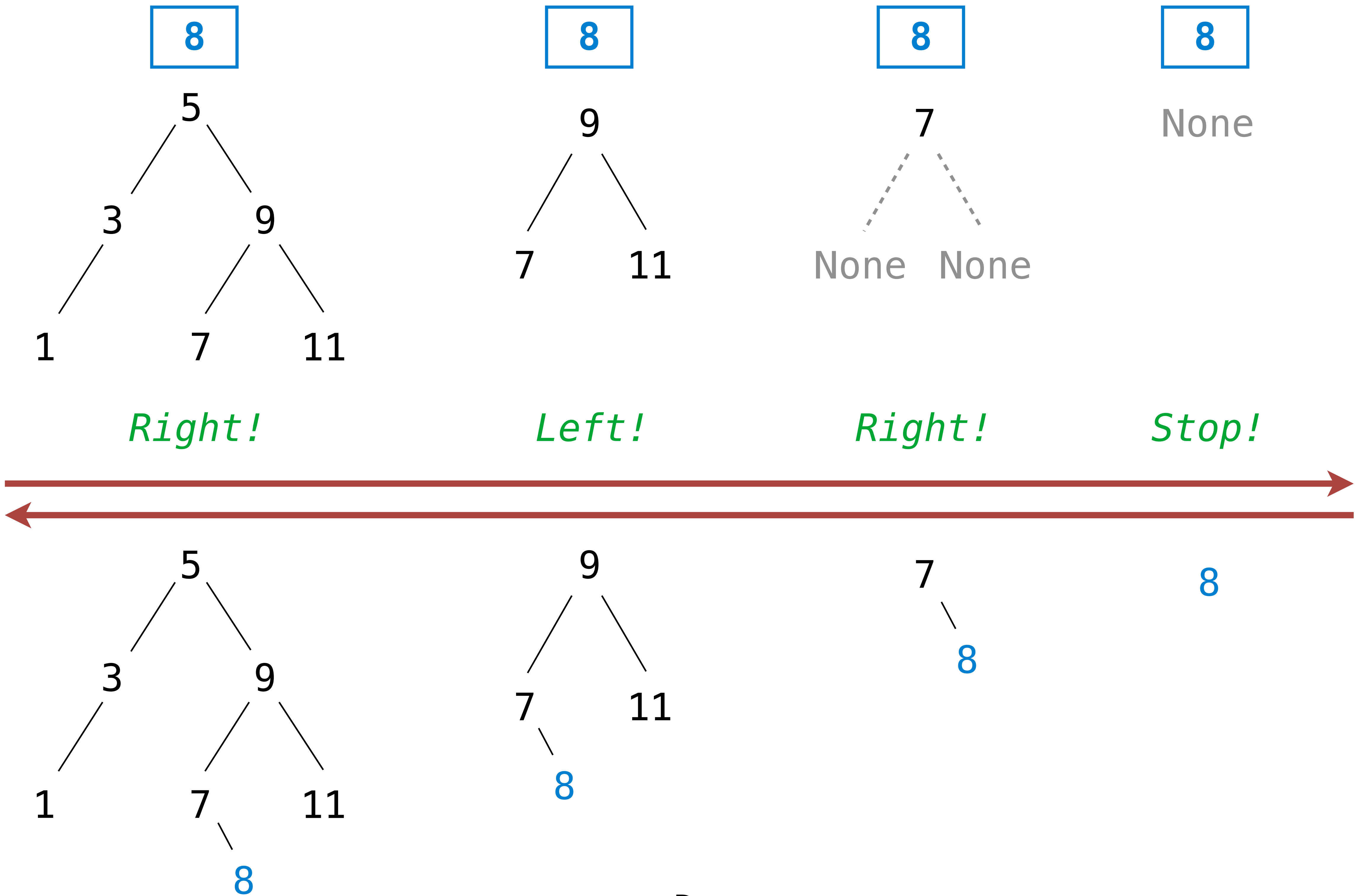


Demo

# What Did I Leave Out?

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Sets as ordered sequences:
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**Good luck!**