61A Lecture 33

18th November, 2011

Last time

Last time

Why is parallel computation important?

Last time

Why is parallel computation important?

What is parallel computation?

Last time

Why is parallel computation important?

What is parallel computation?

Some examples in Python

Last time

Why is parallel computation important?

What is parallel computation?

Some examples in Python

Some problems with parallel computation

Parallel computation terminology

Parallel computation terminology

`Processor`

Parallel computation terminology

Processor

- One of (possibly) many pieces of hardware responsible for executing instructions

Parallel computation terminology

Processor

- One of (possibly) many pieces of hardware responsible for executing instructions

Thread

Parallel computation terminology

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- One of (possibly) many pieces of hardware responsible for executing instructions

Thread

- One of (possibly) many simultaneous sequences of instructions, being executed in a shared memory environment

Parallel computation terminology

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- One of (possibly) many pieces of hardware responsible for executing instructions

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- One of (possibly) many simultaneous sequences of instructions, being executed in a shared memory environment

Shared memory

Parallel computation terminology

Processor

- One of (possibly) many pieces of hardware responsible for executing instructions

Thread

- One of (possibly) many simultaneous sequences of instructions, being executed in a shared memory environment

Shared memory

- The environment in which threads are executed, containing variables that are accessible to all the threads.

Today: dealing with shared memory

Today: dealing with shared memory

"Vulnerable sections" of a program

Today: dealing with shared memory

"Vulnerable sections" of a program

- Critical Sections

Today: dealing with shared memory

"Vulnerable sections" of a program

- Critical Sections
- Atomicity

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Correctness

Today: dealing with shared memory

"Vulnerable sections" of a program

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Correctness

- What does "correctness" mean for parallel computation?

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Protecting vulnerable sections

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- What does "correctness" mean for parallel computation?

Protecting vulnerable sections

- Locks

Today: dealing with shared memory

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Correctness

- What does "correctness" mean for parallel computation?

Protecting vulnerable sections

- Locks
- Semaphores

Today: dealing with shared memory

"Vulnerable sections" of a program

- Critical Sections
- Atomicity

Correctness

- What does "correctness" mean for parallel computation?

Protecting vulnerable sections

- Locks
- Semaphores
- Conditions

Today: dealing with shared memory

"Vulnerable sections" of a program

- Critical Sections
- Atomicity

Correctness

- What does "correctness" mean for parallel computation?

Protecting vulnerable sections

- Locks
- Semaphores
- Conditions

Deadlock

Parallel computing example: bank balance

```
def make_withdraw(balance):
    def withdraw(amount):
        nonlocal balance
        if amount > balance:
            print('Insufficient funds')
        else:
            balance = balance - amount
            print(balance)
    return withdraw
```

Parallel computing example: bank balance

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def make_withdraw(balance):
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```

```
w = make_withdraw(10)
balance = 10
```

Parallel computing example: bank balance

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    return withdraw
```

```
w = make_withdraw(10)
balance = 10
```

```
w(8)
```

```
w(7)
```

Parallel computing example: bank balance

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def make_withdraw(balance):
    def withdraw(amount):
        nonlocal balance
        if amount > balance:
            print('Insufficient funds')
        else:
            balance = balance - amount
            print(balance)
    return withdraw
```

```
w = make_withdraw(10)
balance = 10  2 or 3
```

```
w(8)
```

```
w(7)
```

Parallel computing example: bank balance

```
def make_withdraw(balance):
    def withdraw(amount):
        nonlocal balance
        if amount > balance:
            print('Insufficient funds')
        else:
            balance = balance - amount
            print(balance)
    return withdraw
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```
w = make_withdraw(10)
balance = 10  2 or 3
```

```
w(8)
```

```
w(7)
```

```
print('Insufficient funds')
```

Parallel computing example: bank balance

```
def make_withdraw(balance):
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Parallel computing example: bank balance

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Parallel computing example: bank balance

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w = make_withdraw(10)
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w(7)
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Parallel computing example: bank balance

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        else:
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            print(balance)
    return withdraw
```

```
w = make_withdraw(10)
balance = 10
```

```
w(8)
```

```
read balance: 10
```

```
w(7)
```

Parallel computing example: bank balance

```
def make_withdraw(balance):
    def withdraw(amount):
        nonlocal balance
        if amount > balance:
            print('Insufficient funds')
        else:
            balance = balance - amount
            print(balance)
    return withdraw
```

```
w = make_withdraw(10)
balance = 10
```

```
w(8)
```

```
read balance: 10
read amount: 8
```

```
w(7)
```

```
read balance: 10
```

Parallel computing example: bank balance

```
def make_withdraw(balance):
    def withdraw(amount):
        nonlocal balance
        if amount > balance:
            print('Insufficient funds')
        else:
            balance = balance - amount
            print(balance)
    return withdraw
```

```
w = make_withdraw(10)
balance = 10
```

```
w(8)
```

```
read balance: 10
read amount: 8
8 > 10: False
```

```
w(7)
```

```
read balance: 10
read amount: 7
```

Parallel computing example: bank balance

```
def make_withdraw(balance):
    def withdraw(amount):
        nonlocal balance
        if amount > balance:
            print('Insufficient funds')
        else:
            balance = balance - amount
            print(balance)
    return withdraw
```

```
w = make_withdraw(10)
balance = 10
```

```
w(8)
```

```
read balance: 10
read amount: 8
8 > 10: False
if False
```

```
w(7)
```

```
read balance: 10
read amount: 7
7 > 10: False
```

Parallel computing example: bank balance

```
def make_withdraw(balance):
    def withdraw(amount):
        nonlocal balance
        if amount > balance:
            print('Insufficient funds')
        else:
            balance = balance - amount
            print(balance)
    return withdraw
```

```
w = make_withdraw(10)
balance = 10
```

```
w(8)
```

```
read balance: 10
read amount: 8
8 > 10: False
if False
10 - 8: 2
```

```
w(7)
```

```
read balance: 10
read amount: 7
7 > 10: False
if False
```

Parallel computing example: bank balance

```
def make_withdraw(balance):
    def withdraw(amount):
        nonlocal balance
        if amount > balance:
            print('Insufficient funds')
        else:
            balance = balance - amount
            print(balance)
    return withdraw
```

```
w = make_withdraw(10)
balance = 10
```

```
w(8)
```

```
read balance: 10
read amount: 8
8 > 10: False
if False
10 - 8: 2
write balance -> 2
```

```
w(7)
```

```
read balance: 10
read amount: 7
7 > 10: False
if False
10 - 7: 3
```

Parallel computing example: bank balance

```
def make_withdraw(balance):
    def withdraw(amount):
        nonlocal balance
        if amount > balance:
            print('Insufficient funds')
        else:
            balance = balance - amount
            print(balance)
    return withdraw
```

```
w = make_withdraw(10)
balance = 10 2
```

w(8)

```
read balance: 10
read amount: 8
8 > 10: False
if False
10 - 8: 2
write balance -> 2
```

w(7)

```
read balance: 10
read amount: 7
7 > 10: False
if False
10 - 7: 3
```

Parallel computing example: bank balance

```
def make_withdraw(balance):
    def withdraw(amount):
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        if amount > balance:
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    return withdraw
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```
w = make_withdraw(10)
balance = 10 2
```

w(8)

```
read balance: 10
read amount: 8
8 > 10: False
if False
10 - 8: 2
write balance -> 2
print 2
```

w(7)

```
read balance: 10
read amount: 7
7 > 10: False
if False
10 - 7: 3
write balance -> 3
```

Parallel computing example: bank balance

```
def make_withdraw(balance):
    def withdraw(amount):
        nonlocal balance
        if amount > balance:
            print('Insufficient funds')
        else:
            balance = balance - amount
            print(balance)
    return withdraw
```

```
w = make_withdraw(10)
balance = 10 2 3
```

w(8)

```
read balance: 10
read amount: 8
8 > 10: False
if False
10 - 8: 2
write balance -> 2
print 2
```

w(7)

```
read balance: 10
read amount: 7
7 > 10: False
if False
10 - 7: 3
write balance -> 3
```

Parallel computing example: bank balance

```
def make_withdraw(balance):
    def withdraw(amount):
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        if amount > balance:
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            print(balance)
    return withdraw
```

```
w = make_withdraw(10)
balance = 10 2 3
```

w(8)

```
read balance: 10
read amount: 8
8 > 10: False
if False
10 - 8: 2
write balance -> 2
print 2
```

w(7)

```
read balance: 10
read amount: 7
7 > 10: False
if False
10 - 7: 3
write balance -> 3
print 3
```

Parallel computing example: bank balance

```
def make_withdraw(balance):
    def withdraw(amount):
        nonlocal balance
        if amount > balance:
            print('Insufficient funds')
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            balance = balance - amount
            print(balance)
    return withdraw
```

```
w = make_withdraw(10)
balance = 10 2 3
```

w(8)

```
read balance: 10
read amount: 8
8 > 10: False
if False
10 - 8: 2
write balance -> 2
print 2
```

w(7)

```
read balance: 10
read amount: 7
7 > 10: False
if False
10 - 7: 3
write balance -> 3
print 3
```

$15 withdrawn from a $10 account?
With $3 left? Inconceivable!

Parallel computing example: bank balance

```
def make_withdraw(balance):
    def withdraw(amount):
        nonlocal balance
        if amount > balance:
            print('Insufficient funds')
        else:
            balance = balance - amount
            print(balance)
    return withdraw
```

```
w = make_withdraw(10)
balance = 10  2 or 3
```

```
w(8)
```

```
w(7)
```

```
print('Insufficient funds')
```

Another problem: vector mathematics

$$A = B+C$$

$$V = MxA$$

Vector mathematics

$$B=\begin{pmatrix}2\\0\end{pmatrix}\quad C=\begin{pmatrix}0\\5\end{pmatrix}\quad M=\begin{pmatrix}1 & 2\\1 & 2\end{pmatrix}$$

Vector mathematics

$$A = B+C$$
$$V = MxA$$

$$B=\begin{pmatrix}2\\0\end{pmatrix} \quad C=\begin{pmatrix}0\\5\end{pmatrix} \quad M=\begin{pmatrix}1 & 2\\1 & 2\end{pmatrix}$$

Vector mathematics

$$A = B + C$$
$$V = M \times A$$

$$A=\begin{pmatrix}2\\5\end{pmatrix} \quad V=\begin{pmatrix}12\\12\end{pmatrix} \quad B=\begin{pmatrix}2\\0\end{pmatrix} \quad C=\begin{pmatrix}0\\5\end{pmatrix} \quad M=\begin{pmatrix}1 & 2\\1 & 2\end{pmatrix}$$

Vector mathematics

$$A = B+C$$
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$$A_1 = B_1+C_1$$
$$V_1 = M_1.A$$

$$A_2 = B_2+C_2$$
$$V_2 = M_2.A$$

Vector mathematics

$$A = B+C$$
$$V = MxA$$

$$A=\begin{pmatrix}2\\5\end{pmatrix} \quad V=\begin{pmatrix}12\\12\end{pmatrix} \quad B=\begin{pmatrix}2\\0\end{pmatrix} \quad C=\begin{pmatrix}0\\5\end{pmatrix} \quad M=\begin{pmatrix}1 & 2\\1 & 2\end{pmatrix}$$

$$A_1 = B_1+C_1$$
$$V_1 = M_1.A$$

P1

$$A_2 = B_2+C_2$$
$$V_2 = M_2.A$$

P2

Vector mathematics

$$A = B+C$$
$$V = MxA$$

$$A=\begin{pmatrix}2\\5\end{pmatrix} \quad V=\begin{pmatrix}12\\12\end{pmatrix} \quad B=\begin{pmatrix}2\\0\end{pmatrix} \quad C=\begin{pmatrix}0\\5\end{pmatrix} \quad M=\begin{pmatrix}1 & 2\\1 & 2\end{pmatrix}$$

$$A_1 = B_1+C_1$$
$$V_1 = M_1.A$$

P1

read B1: 2

$$A_2 = B_2+C_2$$
$$V_2 = M_2.A$$

P2

Vector mathematics

$$A = B + C$$
$$V = M \times A$$

$$A=\begin{pmatrix}2\\5\end{pmatrix} \quad V=\begin{pmatrix}12\\12\end{pmatrix} \quad B=\begin{pmatrix}2\\0\end{pmatrix} \quad C=\begin{pmatrix}0\\5\end{pmatrix} \quad M=\begin{pmatrix}1 & 2\\1 & 2\end{pmatrix}$$

$$A_1 = B_1 + C_1$$
$$V_1 = M_1 . A$$

$$A_2 = B_2 + C_2$$
$$V_2 = M_2 . A$$

P1

```
read B1: 2
read C1: 0
```

P2

Vector mathematics

$$A = B + C$$
$$V = M \times A$$

$$A=\begin{pmatrix}2\\5\end{pmatrix} \quad V=\begin{pmatrix}12\\12\end{pmatrix} \quad B=\begin{pmatrix}2\\0\end{pmatrix} \quad C=\begin{pmatrix}0\\5\end{pmatrix} \quad M=\begin{pmatrix}1 & 2\\1 & 2\end{pmatrix}$$

$$A_1 = B_1 + C_1$$
$$V_1 = M_1 . A$$

P1

read B1: 2
read C1: 0
calculate 2+0: 2

$$A_2 = B_2 + C_2$$
$$V_2 = M_2 . A$$

P2

Vector mathematics

$$A = B+C$$
$$V = MxA$$

$$A=\begin{pmatrix}2\\5\end{pmatrix}\quad V=\begin{pmatrix}12\\12\end{pmatrix}\quad B=\begin{pmatrix}2\\0\end{pmatrix}\quad C=\begin{pmatrix}0\\5\end{pmatrix}\quad M=\begin{pmatrix}1 & 2\\1 & 2\end{pmatrix}$$

$$A_1 = B_1+C_1$$
$$V_1 = M_1.A$$

$$A_2 = B_2+C_2$$
$$V_2 = M_2.A$$

P1

```
read B1: 2
read C1: 0
calculate 2+0: 2
write 2 -> A1
```

P2

```
read B2: 0
```

Vector mathematics

$$A = B+C$$
$$V = MxA$$

$$A=\begin{pmatrix}2\\5\end{pmatrix} \quad V=\begin{pmatrix}12\\12\end{pmatrix} \quad B=\begin{pmatrix}2\\0\end{pmatrix} \quad C=\begin{pmatrix}0\\5\end{pmatrix} \quad M=\begin{pmatrix}1 & 2\\1 & 2\end{pmatrix} \quad A=\begin{pmatrix}2\\0\end{pmatrix}$$

$$A_1 = B_1+C_1$$
$$V_1 = M_1.A$$

$$A_2 = B_2+C_2$$
$$V_2 = M_2.A$$

P1

```
read B1: 2
read C1: 0
calculate 2+0: 2
write 2 -> A1
```

P2

```
read B2: 0
```

Vector mathematics

$$A = B+C$$
$$V = MxA$$

$$A=\begin{pmatrix}2\\5\end{pmatrix} \quad V=\begin{pmatrix}12\\12\end{pmatrix} \quad B=\begin{pmatrix}2\\0\end{pmatrix} \quad C=\begin{pmatrix}0\\5\end{pmatrix} \quad M=\begin{pmatrix}1 & 2\\1 & 2\end{pmatrix} \quad A=\begin{pmatrix}2\\0\end{pmatrix}$$

$$A_1 = B_1+C_1$$
$$V_1 = M_1.A$$

$$A_2 = B_2+C_2$$
$$V_2 = M_2.A$$

P1

```
read B1: 2
read C1: 0
calculate 2+0: 2
write 2 -> A1
read M1: (1 2)
```

P2

```
read B2: 0
read C2: 5
```

Vector mathematics

$$A = B + C$$
$$V = M \times A$$

$$A=\begin{pmatrix}2\\5\end{pmatrix} \quad V=\begin{pmatrix}12\\12\end{pmatrix} \quad B=\begin{pmatrix}2\\0\end{pmatrix} \quad C=\begin{pmatrix}0\\5\end{pmatrix} \quad M=\begin{pmatrix}1 & 2\\1 & 2\end{pmatrix} \quad A=\begin{pmatrix}2\\0\end{pmatrix}$$

$$A_1 = B_1 + C_1$$
$$V_1 = M_1 . A$$

P1

```
read B1: 2
read C1: 0
calculate 2+0: 2
write 2 -> A1
read M1: (1 2)
read A: (2 0)
```

$$A_2 = B_2 + C_2$$
$$V_2 = M_2 . A$$

P2

```
read B2: 0
read C2: 5
calculate 5+0: 5
```

Vector mathematics

$$A = B+C$$
$$V = MxA$$

$$A=\begin{pmatrix}2\\5\end{pmatrix} \quad V=\begin{pmatrix}12\\12\end{pmatrix} \quad B=\begin{pmatrix}2\\0\end{pmatrix} \quad C=\begin{pmatrix}0\\5\end{pmatrix} \quad M=\begin{pmatrix}1 & 2\\1 & 2\end{pmatrix} \quad A=\begin{pmatrix}2\\0\end{pmatrix}$$

$$A_1 = B_1+C_1$$
$$V_1 = M_1.A$$

$$A_2 = B_2+C_2$$
$$V_2 = M_2.A$$

P1

```
read B1: 2
read C1: 0
calculate 2+0: 2
write 2 -> A1
read M1: (1 2)
read A: (2 0)
calculate (1 2).(2 0): 2
```

P2

```
read B2: 0
read C2: 5
calculate 5+0: 5
write 5 -> A2
```

Vector mathematics

$$A = B + C$$
$$V = M \times A$$

$$A=\begin{pmatrix}2\\5\end{pmatrix} \quad V=\begin{pmatrix}12\\12\end{pmatrix} \quad B=\begin{pmatrix}2\\0\end{pmatrix} \quad C=\begin{pmatrix}0\\5\end{pmatrix} \quad M=\begin{pmatrix}1 & 2\\1 & 2\end{pmatrix} \quad A=\begin{pmatrix}2\\5\end{pmatrix}$$

$$A_1 = B_1 + C_1$$
$$V_1 = M_1 . A$$

P1

```
read B1: 2
read C1: 0
calculate 2+0: 2
write 2 -> A1
read M1: (1 2)
read A: (2 0)
calculate (1 2).(2 0): 2
```

$$A_2 = B_2 + C_2$$
$$V_2 = M_2 . A$$

P2

```
read B2: 0
read C2: 5
calculate 5+0: 5
write 5 -> A2
```

Vector mathematics

$$A = B + C$$
$$V = M \times A$$

$$A=\begin{pmatrix}2\\5\end{pmatrix} \quad V=\begin{pmatrix}12\\12\end{pmatrix} \quad B=\begin{pmatrix}2\\0\end{pmatrix} \quad C=\begin{pmatrix}0\\5\end{pmatrix} \quad M=\begin{pmatrix}1 & 2\\1 & 2\end{pmatrix} \quad A=\begin{pmatrix}2\\5\end{pmatrix}$$

$$A_1 = B_1 + C_1$$
$$V_1 = M_1 . A$$

$$A_2 = B_2 + C_2$$
$$V_2 = M_2 . A$$

P1

```
read B1: 2
read C1: 0
calculate 2+0: 2
write 2 -> A1
read M1: (1 2)
read A: (2 0)
calculate (1 2).(2 0): 2
write 2 -> V1
```

P2

```
read B2: 0
read C2: 5
calculate 5+0: 5
write 5 -> A2
read M2: (1 2)
```

Vector mathematics

$$A = B + C$$
$$V = M \times A$$

$$A=\begin{pmatrix}2\\5\end{pmatrix} \quad V=\begin{pmatrix}12\\12\end{pmatrix} \quad B=\begin{pmatrix}2\\0\end{pmatrix} \quad C=\begin{pmatrix}0\\5\end{pmatrix} \quad M=\begin{pmatrix}1 & 2\\1 & 2\end{pmatrix} \quad A=\begin{pmatrix}2\\5\end{pmatrix}$$

$$A_1 = B_1 + C_1$$
$$V_1 = M_1 . A$$

P1

```
read B1: 2
read C1: 0
calculate 2+0: 2
write 2 -> A1
read M1: (1 2)
read A: (2 0)
calculate (1 2).(2 0): 2
write 2 -> V1
```

$$A_2 = B_2 + C_2$$
$$V_2 = M_2 . A$$

P2

```
read B2: 0
read C2: 5
calculate 5+0: 5
write 5 -> A2
read M2: (1 2)
read A: (2 5)
```

Vector mathematics

$$A = B + C$$
$$V = M \times A$$

$$A=\begin{pmatrix}2\\5\end{pmatrix} \quad V=\begin{pmatrix}12\\12\end{pmatrix} \quad B=\begin{pmatrix}2\\0\end{pmatrix} \quad C=\begin{pmatrix}0\\5\end{pmatrix} \quad M=\begin{pmatrix}1 & 2\\1 & 2\end{pmatrix} \quad A=\begin{pmatrix}2\\5\end{pmatrix}$$

$$A_1 = B_1 + C_1$$
$$V_1 = M_1 . A$$

P1

```
read B1: 2
read C1: 0
calculate 2+0: 2
write 2 -> A1
read M1: (1 2)
read A: (2 0)
calculate (1 2).(2 0): 2
write 2 -> V1
```

$$A_2 = B_2 + C_2$$
$$V_2 = M_2 . A$$

P2

```
read B2: 0
read C2: 5
calculate 5+0: 5
write 5 -> A2
read M2: (1 2)
read A: (2 5)
calculate (1 2).(2 5):12
```

Vector mathematics

$$A = B + C$$
$$V = M \times A$$

$$A=\begin{pmatrix}2\\5\end{pmatrix} \quad V=\begin{pmatrix}12\\12\end{pmatrix} \quad B=\begin{pmatrix}2\\0\end{pmatrix} \quad C=\begin{pmatrix}0\\5\end{pmatrix} \quad M=\begin{pmatrix}1 & 2\\1 & 2\end{pmatrix} \quad A=\begin{pmatrix}2\\5\end{pmatrix}$$

$$A_1 = B_1 + C_1$$
$$V_1 = M_1 . A$$

P1

```
read B1: 2
read C1: 0
calculate 2+0: 2
write 2 -> A1
read M1: (1 2)
read A: (2 0)
calculate (1 2).(2 0): 2
write 2 -> V1
```

$$A_2 = B_2 + C_2$$
$$V_2 = M_2 . A$$

P2

```
read B2: 0
read C2: 5
calculate 5+0: 5
write 5 -> A2
read M2: (1 2)
read A: (2 5)
calculate (1 2).(2 5):12
write 12 -> V2
```

Vector mathematics

$$A = B+C$$
$$V = M \times A$$

$$A=\begin{pmatrix}2\\5\end{pmatrix} \quad V=\begin{pmatrix}12\\12\end{pmatrix} \quad B=\begin{pmatrix}2\\0\end{pmatrix} \quad C=\begin{pmatrix}0\\5\end{pmatrix} \quad M=\begin{pmatrix}1 & 2\\1 & 2\end{pmatrix} \quad A=\begin{pmatrix}2\\5\end{pmatrix}$$

$$A_1 = B_1+C_1$$
$$V_1 = M_1.A$$

P1

```
read B1: 2
read C1: 0
calculate 2+0: 2
write 2 -> A1
read M1: (1 2)
read A: (2 0)
calculate (1 2).(2 0): 2
write 2 -> V1
```

$$V=\begin{pmatrix}2\\12\end{pmatrix}$$

$$A_2 = B_2+C_2$$
$$V_2 = M_2.A$$

P2

```
read B2: 0
read C2: 5
calculate 5+0: 5
write 5 -> A2
read M2: (1 2)
read A: (2 5)
calculate (1 2).(2 5):12
write 12 -> V2
```

Vector mathematics

$$A = B+C$$

$$V = MxA$$

Vector mathematics

$$A = B+C$$

$$V = MxA$$

Vector mathematics

Step 1

$$A = B+C$$

$$V = MxA$$

Vector mathematics

Step 1

$$A = B+C$$

Step 2

$$V = MxA$$

Vector mathematics

Step 1

$$A = B+C$$

Step 2

$$V = MxA$$

Threads must wait for each other.
Only move on when all have finished previous step.

Correctness

The outcome should *always* be equivalent to some serial ordering of individual steps.

serial ordering: if the threads were executed individually, from start to finish, one after the other instead of in parallel.

Problem 1: inconsistent values

Need ways to make threads wait.

Problem 1: inconsistent values

Inconsistent values

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Problem 1: inconsistent values

Inconsistent values

- A thread reads a value and starts processing

Need ways to make threads wait.

Problem 1: inconsistent values

Inconsistent values

- A thread reads a value and starts processing
- Another thread changes the value

Need ways to make threads wait.

Problem 1: inconsistent values

Inconsistent values

- A thread reads a value and starts processing
- Another thread changes the value
- The first thread’s value is inconsistent and out of date

Need ways to make threads wait.

Problem 1: inconsistent values

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Problem 2: unsynchronized threads

Need ways to make threads wait.

Problem 1: inconsistent values

Inconsistent values

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Problem 2: unsynchronized threads

Unsynchronized threads

Need ways to make threads wait.

Problem 1: inconsistent values

Inconsistent values

- A thread reads a value and starts processing
- Another thread changes the value
- The first thread’s value is inconsistent and out of date

Problem 2: unsynchronized threads

Unsynchronized threads

- Operations is a series of steps

Need ways to make threads wait.

Problem 1: inconsistent values

Inconsistent values

- A thread reads a value and starts processing
- Another thread changes the value
- The first thread’s value is inconsistent and out of date

Problem 2: unsynchronized threads

Unsynchronized threads

- Operations is a series of steps
- Threads must wait until all have finished previous step

Need ways to make threads wait.

Problem 1: inconsistent values

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Inconsistent values

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Inconsistent values

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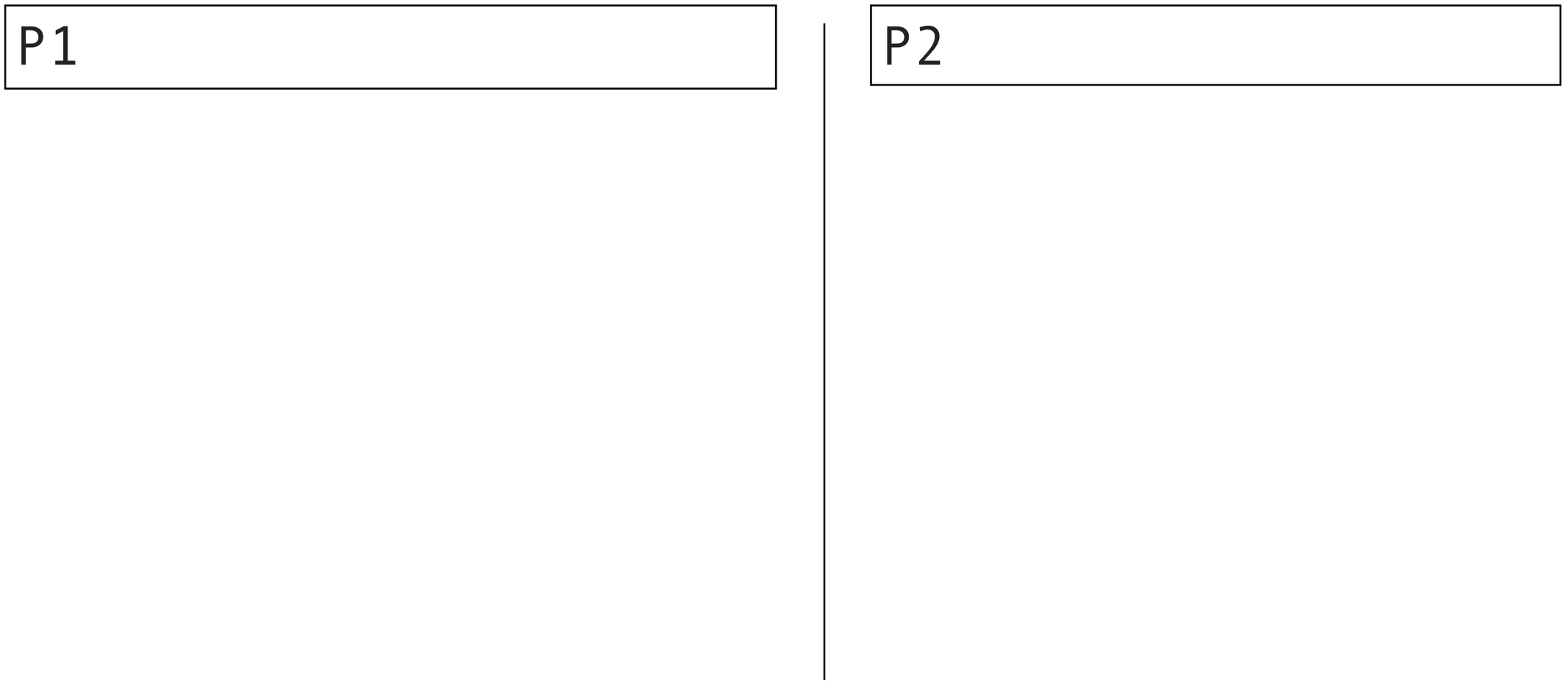

Problem 1: inconsistent values

Inconsistent values

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P1	P2
harmless code	

Problem 1: inconsistent values

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P1	P2
harmless code harmless code	

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P1	P2
harmless code	
harmless code	
modify shared variable	

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P1	P2
harmless code	
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.....	

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P1	P2
harmless code	
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.....	
.....	

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P1	P2
harmless code	
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. . . . .	
. . . . .	
. . . . .	

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P1	P2
harmless code	
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. . . . .	
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. . . . .	

Problem 1: inconsistent values

Inconsistent values

- A thread reads a value and starts processing
- Another thread changes the value
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P1

```
harmless code
harmless code
modify shared variable
.............
.............
.............
.............
write shared variable
```

P2

Problem 1: inconsistent values

Inconsistent values

- A thread reads a value and starts processing
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P1

```
harmless code
harmless code
modify shared variable
.............
.............
.............
.............
write shared variable
```

}

P2

Should not be interrupted by other threads that access same variable

Problem 1: inconsistent values

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- A thread reads a value and starts processing
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- The first thread's value is inconsistent and out of date

P1

```
harmless code
harmless code
modify shared variable
.............
.............
.............
.............
write shared variable
harmless code
```

P2

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.............
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harmless code
harmless code
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Should not be interrupted by other threads that access same variable

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```
harmless code
harmless code
modify shared variable
.............
.............
.............
.............
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harmless code
harmless code
```

P2

Critical Section

Should not be interrupted by other threads that access same variable

Terminology

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“Critical section”

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- A section of code that should not be interrupted

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- Atomic: cannot be broken down into further pieces

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- Atomic: cannot be broken down into further pieces
- Atomic (when applied to code): cannot be interrupted, like a single hardware instruction.

Terminology

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- A section of code that should not be interrupted
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- Atomic: cannot be broken down into further pieces
- Atomic (when applied to code): cannot be interrupted, like a single hardware instruction.
- Atomicity: a guarantee that the code will not be interrupted.

Terminology

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- A section of code that should not be interrupted
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"Atomic" and "Atomicity"

- Atomic: cannot be broken down into further pieces
- Atomic (when applied to code): cannot be interrupted, like a single hardware instruction.
- Atomicity: a guarantee that the code will not be interrupted.

Critical sections need to have atomicity.

Protecting shared state *with shared state*

Protecting shared state *with shared state*

Use shared state to store signals

Protecting shared state *with shared state*

Use shared state to store signals

Signals can indicate:

Protecting shared state *with shared state*

Use shared state to store signals

Signals can indicate:

- A variable is in use

Protecting shared state *with shared state*

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Signals can indicate:

- A variable is in use
- A step is complete (or not)

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Signals can indicate:

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Signals can indicate:

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Signals:

- Locks or mutexes (mutual exclusions)

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Don’t physically protect shared state

Protecting shared state *with shared state*

Use shared state to store signals

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- A step is complete (or not)
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- Whether or not a condition is true

Signals:

- Locks or mutexes (mutual exclusions)
- Semaphores
- Conditions

Don’t physically protect shared state

Convention and shared rules for signals protect shared state.

Protecting shared state *with shared state*

Use shared state to store signals

Signals can indicate:

- A variable is in use
- A step is complete (or not)
- How many threads are using a resource
- Whether or not a condition is true

Signals:

- Locks or mutexes (mutual exclusions)
- Semaphores
- Conditions

Don’t physically protect shared state

Convention and shared rules for signals protect shared state.

- Like traffic signals “protect” an intersection

Locks

Locks

Implemented using real atomic hardware instructions.

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Used to signal that a shared resource is in use.

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acquire()

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- “set” the signal.

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- No other threads will be able to acquire()

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release()

Locks

Implemented using real atomic hardware instructions.

Used to signal that a shared resource is in use.

acquire()

- “set” the signal.
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- They will automatically wait until ...

release()

- “unset” a signal.

Locks

Implemented using real atomic hardware instructions.

Used to signal that a shared resource is in use.

acquire()

- “set” the signal.
- No other threads will be able to acquire()
- They will automatically wait until ...

release()

- “unset” a signal.
- Any one thread that was waiting for acquire() will now succeed

Using locks: bank balance example

```
def make_withdraw(balance):
    def withdraw(amount):
        nonlocal balance
        if amount > balance:
            print('Insufficient funds')
        else:
            balance = balance - amount
            print(balance)
    return withdraw
```

Using locks: bank balance example

```
def make_withdraw(balance):
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```

```
w = make_withdraw(10)
balance = 10
```

Using locks: bank balance example

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```

```
w = make_withdraw(10)
balance = 10
```

```
w(8)
```

```
w(7)
```

Using locks: bank balance example

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def make_withdraw(balance):
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```
w = make_withdraw(10)
balance = 10
```

```
w(8)
```

```
read balance: 10
```

```
w(7)
```

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```

```
w = make_withdraw(10)
balance = 10
```

```
w(8)
```

```
read balance: 10
read amount: 8
```

```
w(7)
```

```
read balance: 10
```

Using locks: bank balance example

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```

```
w = make_withdraw(10)
balance = 10
```

```
w(8)
```

```
read balance: 10
read amount: 8
8 > 10: False
```

```
w(7)
```

```
read balance: 10
read amount: 7
```

Using locks: bank balance example

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def make_withdraw(balance):
    def withdraw(amount):
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        if amount > balance:
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```

```
w = make_withdraw(10)
balance = 10
```

```
w(8)
```

```
read balance: 10
read amount: 8
8 > 10: False
if False
```

```
w(7)
```

```
read balance: 10
read amount: 7
7 > 10: False
```

Using locks: bank balance example

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def make_withdraw(balance):
    def withdraw(amount):
        nonlocal balance
        if amount > balance:
            print('Insufficient funds')
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            print(balance)
    return withdraw
```

```
w = make_withdraw(10)
balance = 10
```

```
w(8)
```

```
read balance: 10
read amount: 8
8 > 10: False
if False
10 - 8: 2
```

```
w(7)
```

```
read balance: 10
read amount: 7
7 > 10: False
if False
```

Using locks: bank balance example

```
def make_withdraw(balance):
    def withdraw(amount):
        nonlocal balance
        if amount > balance:
            print('Insufficient funds')
        else:
            balance = balance - amount
            print(balance)
    return withdraw
```

```
w = make_withdraw(10)
balance = 10
```

```
w(8)
```

```
read balance: 10
read amount: 8
8 > 10: False
if False
10 - 8: 2
write balance -> 2
```

```
w(7)
```

```
read balance: 10
read amount: 7
7 > 10: False
if False
10 - 7: 3
```

Using locks: bank balance example

```
def make_withdraw(balance):
    def withdraw(amount):
        nonlocal balance
        if amount > balance:
            print('Insufficient funds')
        else:
            balance = balance - amount
            print(balance)
    return withdraw
```

```
w = make_withdraw(10)
balance = 10 2
```

```
w(8)
```

```
read balance: 10
read amount: 8
8 > 10: False
if False
10 - 8: 2
write balance -> 2
```

```
w(7)
```

```
read balance: 10
read amount: 7
7 > 10: False
if False
10 - 7: 3
```

Using locks: bank balance example

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def make_withdraw(balance):
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            print(balance)
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```

```
w = make_withdraw(10)
balance = 10 2
```

```
w(8)
```

```
read balance: 10
read amount: 8
8 > 10: False
if False
10 - 8: 2
write balance -> 2
print 2
```

```
w(7)
```

```
read balance: 10
read amount: 7
7 > 10: False
if False
10 - 7: 3
write balance -> 3
```

Using locks: bank balance example

```
def make_withdraw(balance):
    def withdraw(amount):
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        if amount > balance:
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```
w = make_withdraw(10)
balance = 10 2 3
```

w(8)

```
read balance: 10
read amount: 8
8 > 10: False
if False
10 - 8: 2
write balance -> 2
print 2
```

w(7)

```
read balance: 10
read amount: 7
7 > 10: False
if False
10 - 7: 3
write balance -> 3
```

Using locks: bank balance example

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    def withdraw(amount):
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```
w = make_withdraw(10)
balance = 10 2 3
```

```
w(8)
```

```
read balance: 10
read amount: 8
8 > 10: False
if False
10 - 8: 2
write balance -> 2
print 2
```

```
w(7)
```

```
read balance: 10
read amount: 7
7 > 10: False
if False
10 - 7: 3
write balance -> 3
print 3
```

Using locks: bank balance example

```
def make_withdraw(balance):
    def withdraw(amount):
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            print(balance)
    return withdraw
```

critical section

```
w = make_withdraw(10)
balance = 10  2  3
```

```
w(8)
```

```
read balance: 10
read amount: 8
8 > 10: False
if False
10 - 8: 2
write balance -> 2
print 2
```

```
w(7)
```

```
read balance: 10
read amount: 7
7 > 10: False
if False
10 - 7: 3
write balance -> 3
print 3
```

Using locks: bank balance example

```
def make_withdraw(balance):
    def withdraw(amount):
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critical section

Using locks: bank balance example

```
def make_withdraw(balance):
    def withdraw(amount):
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    return withdraw
```

critical section

New code

Using locks: bank balance example

```
def make_withdraw(balance):
    def withdraw(amount):
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        if amount > balance:
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        else:
            balance = balance - amount
            print(balance)
    return withdraw
```

critical section

New code

```
def make_withdraw(balance)
```

Using locks: bank balance example

```
def make_withdraw(balance):
    def withdraw(amount):
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        if amount > balance:
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            print(balance)
    return withdraw
```

critical section

New code

```
def make_withdraw(balance)
    balance_lock = Lock()
```

Using locks: bank balance example

```
def make_withdraw(balance):
    def withdraw(amount):
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critical section

New code

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def make_withdraw(balance)
    balance_lock = Lock()
    def withdraw(amount):
```

Using locks: bank balance example

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def make_withdraw(balance):
    def withdraw(amount):
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New code

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def make_withdraw(balance)
    balance_lock = Lock()
    def withdraw(amount):
        nonlocal balance
```

Using locks: bank balance example

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        else:
            balance = balance - amount
            print(balance)
    return withdraw
```

critical section

New code

```
def make_withdraw(balance)
    balance_lock = Lock()
    def withdraw(amount):
        nonlocal balance
        # try to acquire the lock
```

Using locks: bank balance example

```
def make_withdraw(balance):
    def withdraw(amount):
        nonlocal balance
        if amount > balance:
            print('Insufficient funds')
        else:
            balance = balance - amount
            print(balance)
    return withdraw
```

critical section

New code

```
def make_withdraw(balance)
    balance_lock = Lock()
    def withdraw(amount):
        nonlocal balance
        # try to acquire the lock
        balance_lock.acquire()
```

Using locks: bank balance example

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def make_withdraw(balance)
    balance_lock = Lock()
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        # once successful, enter the critical section
```

Using locks: bank balance example

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        balance_lock.acquire()
        # once successful, enter the critical section
        if amount > balance:
```

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        else:
```

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```
def make_withdraw(balance):
    def withdraw(amount):
        nonlocal balance
        if amount > balance:
            print('Insufficient funds')
        else:
            balance = balance - amount
            print(balance)
    return withdraw
```

critical section

New code

```
def make_withdraw(balance)
    balance_lock = Lock()
    def withdraw(amount):
        nonlocal balance
        # try to acquire the lock
        balance_lock.acquire()
        # once successful, enter the critical section
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```

Using locks: bank balance example

```
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Using locks: bank balance example

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```

Using locks: bank balance example

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def make_withdraw(balance):
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        # once successful, enter the critical section
        if amount > balance:
            print("Insufficient funds")
        else:
            balance = balance - amount
            print(balance)
        # upon exiting the critical section, release the lock
        balance_lock.release()
```

Using locks: bank balance example

Using locks: bank balance example

```
w = make_withdraw(10)
balance = 10
balance_lock = Lock()
```

Using locks: bank balance example

```
w = make_withdraw(10)
balance = 10
balance_lock = Lock()
```

```
w(8)
```

```
w(7)
```

Using locks: bank balance example

```
w = make_withdraw(10)
balance = 10
balance_lock = Lock()
```

```
w(8)
```

P1

```
w(7)
```

P2

Using locks: bank balance example

```
w = make_withdraw(10)
balance = 10
balance_lock = Lock()
```

w(8)	w(7)
P1	P2
acquire balance_lock: ok	

Using locks: bank balance example

```
w = make_withdraw(10)
balance = 10
balance_lock = Lock()
```
acquired by p1

```
w(8)
```

P1

```
acquire balance_lock: ok
```

```
w(7)
```

P2

Using locks: bank balance example

```
w = make_withdraw(10)
balance = 10
balance_lock = Lock()
```
acquired by p1

w(8)

P1

```
acquire balance_lock: ok
read balance: 10
```

w(7)

P2

```
acquire balance_lock: wait
```

Using locks: bank balance example

```
w = make_withdraw(10)
balance = 10
balance_lock = Lock()  acquired by p1
```

`w(8)`

P1

```
acquire balance_lock: ok
read balance: 10
read amount: 8
```

`w(7)`

P2

```
acquire balance_lock: wait
wait
```

Using locks: bank balance example

```
w = make_withdraw(10)
balance = 10
balance_lock = Lock()  acquired by p1
```

w(8)

P1

```
acquire balance_lock: ok
read balance: 10
read amount: 8
8 > 10: False
```

w(7)

P2

```
acquire balance_lock: wait
wait
wait
```

Using locks: bank balance example

```
w = make_withdraw(10)
balance = 10
balance_lock = Lock()  acquired by p1
```

w(8)

P1

```
acquire balance_lock: ok
read balance: 10
read amount: 8
8 > 10: False
if False
```

w(7)

P2

```
acquire balance_lock: wait
wait
wait
wait
```

Using locks: bank balance example

```
w = make_withdraw(10)
balance = 10
balance_lock = Lock()  acquired by p1
```

`w(8)`

P1

```
acquire balance_lock: ok
read balance: 10
read amount: 8
8 > 10: False
if False
10 - 8: 2
```

`w(7)`

P2

```
acquire balance_lock: wait
wait
wait
wait
wait
```

Using locks: bank balance example

```
w = make_withdraw(10)
balance = 10
balance_lock = Lock()  acquired by p1
```

w(8)

P1

```
acquire balance_lock: ok
read balance: 10
read amount: 8
8 > 10: False
if False
10 - 8: 2
write balance -> 2
```

w(7)

P2

```
acquire balance_lock: wait
wait
wait
wait
wait
wait
```

Using locks: bank balance example

```
w = make_withdraw(10)
balance = 10
balance_lock = Lock()  acquired by p1
```

w(8)

P1

```
acquire balance_lock: ok
read balance: 10
read amount: 8
8 > 10: False
if False
10 - 8: 2
write balance -> 2
print 2
```

w(7)

P2

```
acquire balance_lock: wait
wait
wait
wait
wait
wait
wait
```

Using locks: bank balance example

```
w = make_withdraw(10)
balance = 10
balance_lock = Lock()  acquired by p1
```

w(8)

P1

```
acquire balance_lock: ok
read balance: 10
read amount: 8
8 > 10: False
if False
10 - 8: 2
write balance -> 2
print 2
release balance_lock
```

w(7)

P2

```
acquire balance_lock: wait
wait
wait
wait
wait
wait
wait
wait
```

Using locks: bank balance example

```
w = make_withdraw(10)
balance = 10
balance_lock = Lock()
```

w(8)

P1

```
acquire balance_lock: ok
read balance: 10
read amount: 8
8 > 10: False
if False
10 - 8: 2
write balance -> 2
print 2
release balance_lock
```

w(7)

P2

```
acquire balance_lock: wait
wait
wait
wait
wait
wait
wait
wait
```

Using locks: bank balance example

```
w = make_withdraw(10)
balance = 10
balance_lock = Lock()
```

w(8)

P1

```
acquire balance_lock: ok
read balance: 10
read amount: 8
8 > 10: False
if False
10 - 8: 2
write balance -> 2
print 2
release balance_lock
```

w(7)

P2

```
acquire balance_lock: wait
wait
wait
wait
wait
wait
wait
wait
acquire balance_lock:ok
```

Using locks: bank balance example

```
w = make_withdraw(10)
balance = 10
balance_lock = Lock()
```
acquired by p2

w(8)

P1

```
acquire balance_lock: ok
read balance: 10
read amount: 8
8 > 10: False
if False
10 - 8: 2
write balance -> 2
print 2
release balance_lock
```

w(7)

P2

```
acquire balance_lock: wait
wait
wait
wait
wait
wait
wait
wait
acquire balance_lock:ok
```

Using locks: bank balance example

```
w = make_withdraw(10)
balance = 10
balance_lock = Lock()
```
acquired by p2

w(8)

P1

```
acquire balance_lock: ok
read balance: 10
read amount: 8
8 > 10: False
if False
10 - 8: 2
write balance -> 2
print 2
release balance_lock
```

w(7)

P2

```
acquire balance_lock: wait
wait
wait
wait
wait
wait
wait
wait
acquire balance_lock:ok
read balance: 2
```

Using locks: bank balance example

```
w = make_withdraw(10)
balance = 10
balance_lock = Lock()
```
acquired by p2

w(8)

P1

```
acquire balance_lock: ok
read balance: 10
read amount: 8
8 > 10: False
if False
10 - 8: 2
write balance -> 2
print 2
release balance_lock
```

w(7)

P2

```
acquire balance_lock: wait
wait
wait
wait
wait
wait
wait
wait
acquire balance_lock:ok
read balance: 2
read amount: 7
```

Using locks: bank balance example

```
w = make_withdraw(10)
balance = 10
balance_lock = Lock()
```
acquired by p2

w(8)

P1

```
acquire balance_lock: ok
read balance: 10
read amount: 8
8 > 10: False
if False
10 - 8: 2
write balance -> 2
print 2
release balance_lock
```

w(7)

P2

```
acquire balance_lock: wait
wait
wait
wait
wait
wait
wait
wait
acquire balance_lock:ok
read balance: 2
read amount: 7
7 > 2: True
```

Using locks: bank balance example

```
w = make_withdraw(10)
balance = 10
balance_lock = Lock()
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acquired by p2

w(8)

P1

```
acquire balance_lock: ok
read balance: 10
read amount: 8
8 > 10: False
if False
10 - 8: 2
write balance -> 2
print 2
release balance_lock
```

w(7)

P2

```
acquire balance_lock: wait
wait
wait
wait
wait
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wait
wait
acquire balance_lock:ok
read balance: 2
read amount: 7
7 > 2: True
if True
```

Using locks: bank balance example

```
w = make_withdraw(10)
balance = 10
balance_lock = Lock()
```
acquired by p2

w(8)

P1

```
acquire balance_lock: ok
read balance: 10
read amount: 8
8 > 10: False
if False
10 - 8: 2
write balance -> 2
print 2
release balance_lock
```

w(7)

P2

```
acquire balance_lock: wait
wait
wait
wait
wait
wait
wait
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acquire balance_lock:ok
read balance: 2
read amount: 7
7 > 2: True
if True
print 'Insufficient funds'
```

Using locks: bank balance example

```
w = make_withdraw(10)
balance = 10
balance_lock = Lock()
```
acquired by p2

w(8)

P1

```
acquire balance_lock: ok
read balance: 10
read amount: 8
8 > 10: False
if False
10 - 8: 2
write balance -> 2
print 2
release balance_lock
```

w(7)

P2

```
acquire balance_lock: wait
wait
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read balance: 2
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7 > 2: True
if True
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Using locks: bank balance example

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```

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```
acquire balance_lock: ok
read balance: 10
read amount: 8
8 > 10: False
if False
10 - 8: 2
write balance -> 2
print 2
release balance_lock
```

w(7)

P2

```
acquire balance_lock: wait
wait
wait
wait
wait
wait
wait
wait
acquire balance_lock:ok
read balance: 2
read amount: 7
7 > 2: True
if True
print 'Insufficient funds'
release balance_lock
```

Quiz: does this solution enforce correctness?

```
def make_withdraw(balance)
        balance_lock = Lock()
        def withdraw(amount):
            nonlocal balance
            # try to acquire the lock
            balance_lock.acquire()
            # once successful, enter the critical section
            if amount > balance:
                print("Insufficient funds")
            else:
                balance = balance - amount
                print(balance)
        # upon exiting the critical section, release the lock
        balance_lock.release()
```

Answer: yes

```
def make_withdraw(balance)
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        def withdraw(amount):
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            balance_lock.acquire()
            # once successful, enter the critical section
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           balance_lock.release()
```

No two processes can be in the critical section at the same time.

Answer: yes

```
def make_withdraw(balance)
        balance_lock = Lock()
        def withdraw(amount):
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```

No two processes can be in the critical section at the same time.

Whichever gets to balance_lock.acquire() first gets to finish.

Answer: yes

```
def make_withdraw(balance)
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No two processes can be in the critical section at the same time.

Whichever gets to balance_lock.acquire() first gets to finish.

All others have to wait until it’s finished.

Answer: yes

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        balance_lock.release()
```

important, allows others to proceed

No two processes can be in the critical section at the same time.

Whichever gets to balance_lock.acquire() first gets to finish.

All others have to wait until it's finished.

Answer: yes

remember: always release your locks.

```
def make_withdraw(balance)
        balance_lock = Lock()
        def withdraw(amount):
            nonlocal balance
            # try to acquire the lock
            balance_lock.acquire()
            # once successful, enter the critical section
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important, allows others to proceed

No two processes can be in the critical section at the same time.

Whichever gets to balance_lock.acquire() first gets to finish.

All others have to wait until it's finished.

Semaphores

Semaphores

Used to protect access to limited resources

Each has a limit, N

Can be acquire()'d N times

After that, processes trying to acquire() automatically wait

Until another process release()'s

Semaphores example: database

A database that can only support 2 connections at a time.

Semaphores example: database

A database that can only support 2 connections at a time.

```
# set up the semaphore
```

Semaphores example: database

A database that can only support 2 connections at a time.

```
# set up the semaphore
db_semaphore = Semaphore(2)
```

Semaphores example: database

A database that can only support 2 connections at a time.

```
# set up the semaphore
db_semaphore = Semaphore(2)

def insert(data):
```

Semaphores example: database

A database that can only support 2 connections at a time.

```
# set up the semaphore
db_semaphore = Semaphore(2)

def insert(data):
    # try to acquire the semaphore
```

Semaphores example: database

A database that can only support 2 connections at a time.

```
# set up the semaphore
db_semaphore = Semaphore(2)

def insert(data):
    # try to acquire the semaphore
    db_semaphore.acquire()
```

Semaphores example: database

A database that can only support 2 connections at a time.

```
# set up the semaphore
db_semaphore = Semaphore(2)

def insert(data):
    # try to acquire the semaphore
    db_semaphore.acquire()
    # if successful, proceed
```

Semaphores example: database

A database that can only support 2 connections at a time.

```
# set up the semaphore
db_semaphore = Semaphore(2)

def insert(data):
    # try to acquire the semaphore
    db_semaphore.acquire()
    # if successful, proceed
    database.insert(data)
```

Semaphores example: database

A database that can only support 2 connections at a time.

```
# set up the semaphore
db_semaphore = Semaphore(2)

def insert(data):
# try to acquire the semaphore
db_semaphore.acquire()
# if successful, proceed
database.insert(data)
#release the semaphore
```

Semaphores example: database

A database that can only support 2 connections at a time.

```
# set up the semaphore
db_semaphore = Semaphore(2)

def insert(data):
# try to acquire the semaphore
db_semaphore.acquire()
# if successful, proceed
database.insert(data)
#release the semaphore
db_semaphore.release()
```

Example: database

```
db_semaphore = Semaphore(2)

def insert(data):
    db_semaphore.acquire()
    database.insert(data)
    db_semaphore.release()
```

Example: database

```
db_semaphore = Semaphore(2)

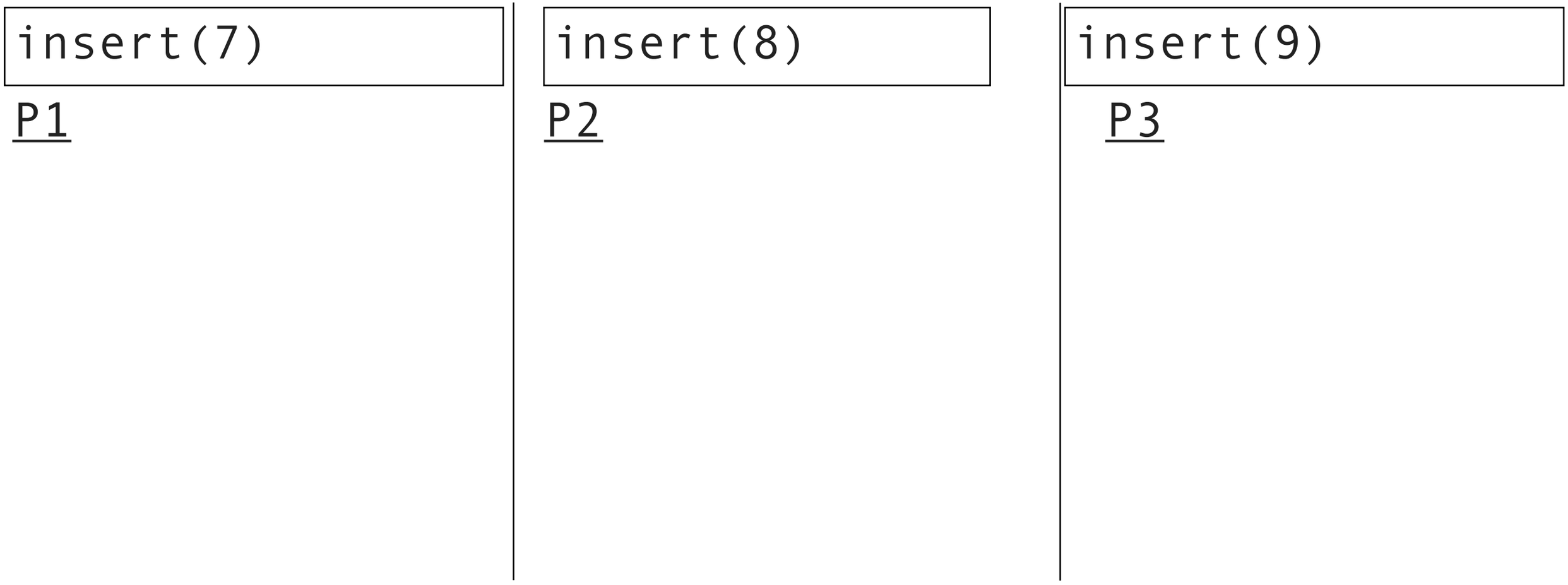
def insert(data):
    db_semaphore.acquire()
    database.insert(data)
    db_semaphore.release()
```

insert(7)	insert(8)	insert(9)

Example: database

```
db_semaphore = Semaphore(2)

def insert(data):
    db_semaphore.acquire()
    database.insert(data)
    db_semaphore.release()
```



Example: database

```
db_semaphore = Semaphore(2)

def insert(data):
    db_semaphore.acquire()
    database.insert(data)
    db_semaphore.release()
```

insert(7)	insert(8)	insert(9)
P1	P2	P3
acquire db_semaphore: ok	acquire db_semaphore: wait	acquire db_semaphore: ok

Example: database

```
db_semaphore = Semaphore(2)

def insert(data):
    db_semaphore.acquire()
    database.insert(data)
    db_semaphore.release()
```

insert(7)	insert(8)	insert(9)
P1	P2	P3
acquire db_semaphore: ok read data: 7	acquire db_semaphore: wait wait	acquire db_semaphore: ok read data: 9

Example: database

```
db_semaphore = Semaphore(2)

def insert(data):
    db_semaphore.acquire()
    database.insert(data)
    db_semaphore.release()
```

insert(7)	insert(8)	insert(9)
P1	P2	P3
acquire db_semaphore: ok read data: 7 read global database	acquire db_semaphore: wait wait wait	acquire db_semaphore: ok read data: 9

Example: database

```
db_semaphore = Semaphore(2)

def insert(data):
    db_semaphore.acquire()
    database.insert(data)
    db_semaphore.release()
```

insert(7)	insert(8)	insert(9)
P1	P2	P3
acquire db_semaphore: ok read data: 7 read global database insert 7 into database	acquire db_semaphore: wait wait wait wait	acquire db_semaphore: ok read data: 9 read global database

Example: database

```
db_semaphore = Semaphore(2)

def insert(data):
    db_semaphore.acquire()
    database.insert(data)
    db_semaphore.release()
```

insert(7)	insert(8)	insert(9)
P1	P2	P3
acquire db_semaphore: ok	acquire db_semaphore: wait	acquire db_semaphore: ok
read data: 7	wait	read data: 9
read global database	wait	
insert 7 into database	wait	read global database
release db_semaphore: ok	acquire db_semaphore: ok	insert 9 into database

Example: database

```
db_semaphore = Semaphore(2)

def insert(data):
    db_semaphore.acquire()
    database.insert(data)
    db_semaphore.release()
```

insert(7)	insert(8)	insert(9)
P1	P2	P3
acquire db_semaphore: ok read data: 7 read global database insert 7 into database release db_semaphore: ok	acquire db_semaphore: wait wait wait wait acquire db_semaphore: ok read data: 8	acquire db_semaphore: ok read data: 9 read global database insert 9 into database release db_semaphore: ok

Example: database

```
db_semaphore = Semaphore(2)

def insert(data):
    db_semaphore.acquire()
    database.insert(data)
    db_semaphore.release()
```

insert(7)	insert(8)	insert(9)
P1	P2	P3
acquire db_semaphore: ok read data: 7 read global database insert 7 into database release db_semaphore: ok	acquire db_semaphore: wait wait wait wait acquire db_semaphore: ok read data: 8 read global database	acquire db_semaphore: ok read data: 9 read global database insert 9 into database release db_semaphore: ok

Example: database

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db_semaphore = Semaphore(2)

def insert(data):
    db_semaphore.acquire()
    database.insert(data)
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```

insert(7)	insert(8)	insert(9)
P1	P2	P3
acquire db_semaphore: ok read data: 7 read global database insert 7 into database release db_semaphore: ok	acquire db_semaphore: wait wait wait wait acquire db_semaphore: ok read data: 8 read global database insert 8 into database	acquire db_semaphore: ok read data: 9 read global database insert 9 into database release db_semaphore: ok

Example: database

```
db_semaphore = Semaphore(2)

def insert(data):
    db_semaphore.acquire()
    database.insert(data)
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```

insert(7)	insert(8)	insert(9)
P1	P2	P3
acquire db_semaphore: ok read data: 7 read global database insert 7 into database release db_semaphore: ok	acquire db_semaphore: wait wait wait wait acquire db_semaphore: ok read data: 8 read global database insert 8 into database release db_semaphore: ok	acquire db_semaphore: ok read data: 9 read global database insert 9 into database release db_semaphore: ok

Conditions

Conditions are signals used to coordinate multiple processes

Processes can wait() on a condition

Other processes can notify() processes waiting for a condition.

Conditions example: vector mathematics

```
A = B+C
V = MxA
```

Conditions example: vector mathematics

```
step1_finished = 0
```

$$A = B+C$$

$$V = MxA$$

Conditions example: vector mathematics

```
step1_finished = 0
start_step2 = Condition()
```

$$A = B+C$$

$$V = MxA$$

Conditions example: vector mathematics

```
step1_finished = 0
start_step2 = Condition()

def do_step_1(index):
```

$$A = B+C$$

$$V = MxA$$

Conditions example: vector mathematics

```
step1_finished = 0
start_step2 = Condition()

def do_step_1(index):
  A[index] = B[index] + C[index]
```

$$A = B+C$$

$$V = MxA$$

Conditions example: vector mathematics

```
step1_finished = 0
start_step2 = Condition()

def do_step_1(index):
  A[index] = B[index] + C[index]
  start_step2.acquire()
```

$$A = B+C$$

$$V = MxA$$

Conditions example: vector mathematics

```
step1_finished = 0
start_step2 = Condition()

def do_step_1(index):
    A[index] = B[index] + C[index]
    start_step2.acquire()
    step1_finished += 1
```

$$A = B+C$$

$$V = MxA$$

Conditions example: vector mathematics

$$A = B+C$$

$$V = MxA$$

```
step1_finished = 0
start_step2 = Condition()

def do_step_1(index):
    A[index] = B[index] + C[index]
    start_step2.acquire()
    step1_finished += 1
    if(step1_finished == 2):
```

Conditions example: vector mathematics

$$A = B+C$$

$$V = MxA$$

```
step1_finished = 0
start_step2 = Condition()

def do_step_1(index):
    A[index] = B[index] + C[index]
    start_step2.acquire()
    step1_finished += 1
    if(step1_finished == 2):
            start_step2.notifyAll()
```

Conditions example: vector mathematics

$$A = B+C$$

$$V = MxA$$

```
step1_finished = 0
start_step2 = Condition()

def do_step_1(index):
    A[index] = B[index] + C[index]
    start_step2.acquire()
    step1_finished += 1
    if(step1_finished == 2):
            start_step2.notifyAll()
    start_step2.release()
```

Conditions example: vector mathematics

$$A = B+C$$
$$V = MxA$$

```
step1_finished = 0
start_step2 = Condition()

def do_step_1(index):
    A[index] = B[index] + C[index]
    start_step2.acquire()
    step1_finished += 1
    if(step1_finished == 2):
            start_step2.notifyAll()
    start_step2.release()

def do_step_2(index):
```

Conditions example: vector mathematics

$$A = B+C$$

$$V = MxA$$

```
step1_finished = 0
start_step2 = Condition()

def do_step_1(index):
   A[index] = B[index] + C[index]
   start_step2.acquire()
   step1_finished += 1
   if(step1_finished == 2):
          start_step2.notifyAll()
   start_step2.release()

def do_step_2(index):
          start_step2.wait()
```

Conditions example: vector mathematics

$$A = B+C$$
$$V = MxA$$

```
step1_finished = 0
start_step2 = Condition()

def do_step_1(index):
   A[index] = B[index] + C[index]
   start_step2.acquire()
   step1_finished += 1
   if(step1_finished == 2):
          start_step2.notifyAll()
   start_step2.release()

def do_step_2(index):
          start_step2.wait()
          V[index] = M[index] . A
```

Conditions example: vector mathematics

```
step1_finished=0

start_step2 = Condition()
```

$$B=\begin{pmatrix}2\\0\end{pmatrix}\quad C=\begin{pmatrix}0\\5\end{pmatrix}\quad M=\begin{pmatrix}1 & 2\\1 & 2\end{pmatrix}$$

Conditions example: vector mathematics

```
step1_finished=0
start_step2 = Condition()
```

$$B=\begin{pmatrix}2\\0\end{pmatrix}\quad C=\begin{pmatrix}0\\5\end{pmatrix}\quad M=\begin{pmatrix}1 & 2\\1 & 2\end{pmatrix}$$

$$A_1 = B_1+C_1$$
$$V_1 = M_1.A$$

$$A_2 = B_2+C_2$$
$$V_2 = M_2.A$$

Conditions example: vector mathematics

```
step1_finished=0
start_step2 = Condition()
```

$$B=\begin{pmatrix}2\\0\end{pmatrix}\quad C=\begin{pmatrix}0\\5\end{pmatrix}\quad M=\begin{pmatrix}1 & 2\\1 & 2\end{pmatrix}$$

$$A_1 = B_1+C_1$$
$$V_1 = M_1.A$$

P1

$$A_2 = B_2+C_2$$
$$V_2 = M_2.A$$

P2

Conditions example: vector mathematics

```
step1_finished=0
start_step2 = Condition()
```

$B = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$ $C = \begin{pmatrix} 0 \\ 5 \end{pmatrix}$ $M = \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix}$

$$A_1 = B_1 + C_1$$
$$V_1 = M_1 . A$$

P1

```
read B1: 2
```

$$A_2 = B_2 + C_2$$
$$V_2 = M_2 . A$$

P2

Conditions example: vector mathematics

```
step1_finished=0
start_step2 = Condition()
```

$B = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$ $C = \begin{pmatrix} 0 \\ 5 \end{pmatrix}$ $M = \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix}$

$$A_1 = B_1 + C_1$$
$$V_1 = M_1 . A$$

P1

```
read B1: 2
read C1: 0
```

$$A_2 = B_2 + C_2$$
$$V_2 = M_2 . A$$

P2

Conditions example: vector mathematics

```
step1_finished=0
start_step2 = Condition()
```

$$B=\begin{pmatrix}2\\0\end{pmatrix}\quad C=\begin{pmatrix}0\\5\end{pmatrix}\quad M=\begin{pmatrix}1 & 2\\1 & 2\end{pmatrix}$$

$$A_1 = B_1+C_1$$
$$V_1 = M_1.A$$

P1

```
read B1: 2
read C1: 0
calculate 2+0: 2
```

$$A_2 = B_2+C_2$$
$$V_2 = M_2.A$$

P2

Conditions example: vector mathematics

```
step1_finished=0
start_step2 = Condition()
```

$B = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$ $C = \begin{pmatrix} 0 \\ 5 \end{pmatrix}$ $M = \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix}$

$$A_1 = B_1 + C_1$$
$$V_1 = M_1 . A$$

P1

```
read B1: 2
read C1: 0
calculate 2+0: 2
write 2 -> A1
```

$$A_2 = B_2 + C_2$$
$$V_2 = M_2 . A$$

P2

```
read B2: 0
```

Conditions example: vector mathematics

```
step1_finished=0
start_step2 = Condition()
```

$B=\begin{pmatrix}2\\0\end{pmatrix}$ $C=\begin{pmatrix}0\\5\end{pmatrix}$ $M=\begin{pmatrix}1 & 2\\1 & 2\end{pmatrix}$ $A=\begin{pmatrix}2\\0\end{pmatrix}$

$$A_1 = B_1+C_1$$
$$V_1 = M_1.A$$

P1

```
read B1: 2
read C1: 0
calculate 2+0: 2
write 2 -> A1
```

$$A_2 = B_2+C_2$$
$$V_2 = M_2.A$$

P2

```
read B2: 0
```

Conditions example: vector mathematics

```
step1_finished=0
start_step2 = Condition()
```

$$B=\begin{pmatrix}2\\0\end{pmatrix}\quad C=\begin{pmatrix}0\\5\end{pmatrix}\quad M=\begin{pmatrix}1 & 2\\1 & 2\end{pmatrix}\quad A=\begin{pmatrix}2\\0\end{pmatrix}$$

$$A_1 = B_1+C_1$$
$$V_1 = M_1.A$$

P1

```
read B1: 2
read C1: 0
calculate 2+0: 2
write 2 -> A1
acquire start_step2: ok
```

$$A_2 = B_2+C_2$$
$$V_2 = M_2.A$$

P2

```
read B2: 0
read C2: 0
```

Conditions example: vector mathematics

```
step1_finished=0
start_step2 = Condition()
```

$$B=\begin{pmatrix}2\\0\end{pmatrix}\quad C=\begin{pmatrix}0\\5\end{pmatrix}\quad M=\begin{pmatrix}1 & 2\\1 & 2\end{pmatrix}\quad A=\begin{pmatrix}2\\0\end{pmatrix}$$

$$A_1 = B_1+C_1$$
$$V_1 = M_1.A$$

P1

```
read B1: 2
read C1: 0
calculate 2+0: 2
write 2 -> A1
acquire start_step2: ok
write 1  -> step1_finished
```

$$A_2 = B_2+C_2$$
$$V_2 = M_2.A$$

P2

```
read B2: 0
read C2: 0
calculate 5+0: 5
```

Conditions example: vector mathematics

```
step1_finished=1
start_step2 = Condition()
```

$$B=\begin{pmatrix}2\\0\end{pmatrix}\quad C=\begin{pmatrix}0\\5\end{pmatrix}\quad M=\begin{pmatrix}1 & 2\\1 & 2\end{pmatrix}\quad A=\begin{pmatrix}2\\0\end{pmatrix}$$

$$A_1 = B_1+C_1$$
$$V_1 = M_1.A$$

P1

```
read B1: 2
read C1: 0
calculate 2+0: 2
write 2 -> A1
acquire start_step2: ok
write 1  -> step1_finished
```

$$A_2 = B_2+C_2$$
$$V_2 = M_2.A$$

P2

```
read B2: 0
read C2: 0
calculate 5+0: 5
```

Conditions example: vector mathematics

```
step1_finished=1
start_step2 = Condition()
```

$$B=\begin{pmatrix}2\\0\end{pmatrix}\quad C=\begin{pmatrix}0\\5\end{pmatrix}\quad M=\begin{pmatrix}1 & 2\\1 & 2\end{pmatrix}\quad A=\begin{pmatrix}2\\0\end{pmatrix}$$

$$A_1 = B_1+C_1$$
$$V_1 = M_1.A$$

P1

```
read B1: 2
read C1: 0
calculate 2+0: 2
write 2 -> A1
acquire start_step2: ok
write 1  -> step1_finished
step1_finished == 2: false
```

$$A_2 = B_2+C_2$$
$$V_2 = M_2.A$$

P2

```
read B2: 0
read C2: 0
calculate 5+0: 5
write 5-> A2
```

Conditions example: vector mathematics

```
step1_finished=1
start_step2 = Condition()
```

$$B=\begin{pmatrix}2\\0\end{pmatrix}\quad C=\begin{pmatrix}0\\5\end{pmatrix}\quad M=\begin{pmatrix}1 & 2\\1 & 2\end{pmatrix}\quad A=\begin{pmatrix}2\\5\end{pmatrix}$$

$$A_1 = B_1+C_1$$
$$V_1 = M_1.A$$

P1

```
read B1: 2
read C1: 0
calculate 2+0: 2
write 2 -> A1
acquire start_step2: ok
write 1  -> step1_finished
step1_finished == 2: false
```

$$A_2 = B_2+C_2$$
$$V_2 = M_2.A$$

P2

```
read B2: 0
read C2: 0
calculate 5+0: 5
write 5-> A2
```

Conditions example: vector mathematics

```
step1_finished=1
start_step2 = Condition()
```

$$B=\begin{pmatrix}2\\0\end{pmatrix}\quad C=\begin{pmatrix}0\\5\end{pmatrix}\quad M=\begin{pmatrix}1 & 2\\1 & 2\end{pmatrix}\quad A=\begin{pmatrix}2\\5\end{pmatrix}$$

$$A_1 = B_1+C_1$$
$$V_1 = M_1.A$$

P1

```
read B1: 2
read C1: 0
calculate 2+0: 2
write 2 -> A1
acquire start_step2: ok
write 1  -> step1_finished
step1_finished == 2: false
release start_step2: ok
```

$$A_2 = B_2+C_2$$
$$V_2 = M_2.A$$

P2

```
read B2: 0
read C2: 0
calculate 5+0: 5
write 5-> A2
acquire start_step2: ok
```

Conditions example: vector mathematics

```
step1_finished=1
start_step2 = Condition()
```

$$B=\begin{pmatrix}2\\0\end{pmatrix}\quad C=\begin{pmatrix}0\\5\end{pmatrix}\quad M=\begin{pmatrix}1 & 2\\1 & 2\end{pmatrix}\quad A=\begin{pmatrix}2\\5\end{pmatrix}$$

$$A_1 = B_1+C_1$$
$$V_1 = M_1.A$$

P1

```
read B1: 2
read C1: 0
calculate 2+0: 2
write 2 -> A1
acquire start_step2: ok
write 1  -> step1_finished
step1_finished == 2: false
release start_step2: ok
start_step2: wait
```

$$A_2 = B_2+C_2$$
$$V_2 = M_2.A$$

P2

```
read B2: 0
read C2: 0
calculate 5+0: 5
write 5-> A2
acquire start_step2: ok
write 2-> step1_finished
```

Conditions example: vector mathematics

```
step1_finished=2

start_step2 = Condition()
```

$$B=\begin{pmatrix}2\\0\end{pmatrix}\quad C=\begin{pmatrix}0\\5\end{pmatrix}\quad M=\begin{pmatrix}1 & 2\\1 & 2\end{pmatrix}\quad A=\begin{pmatrix}2\\5\end{pmatrix}$$

$$A_1 = B_1+C_1$$
$$V_1 = M_1.A$$

P1

```
read B1: 2
read C1: 0
calculate 2+0: 2
write 2 -> A1
acquire start_step2: ok
write 1  -> step1_finished
step1_finished == 2: false
release start_step2: ok
start_step2: wait
```

$$A_2 = B_2+C_2$$
$$V_2 = M_2.A$$

P2

```
read B2: 0
read C2: 0
calculate 5+0: 5
write 5-> A2
acquire start_step2: ok
write 2-> step1_finished
```

Conditions example: vector mathematics

```
step1_finished=2

start_step2 = Condition()
```

$$B=\begin{pmatrix}2\\0\end{pmatrix}\quad C=\begin{pmatrix}0\\5\end{pmatrix}\quad M=\begin{pmatrix}1 & 2\\1 & 2\end{pmatrix}\quad A=\begin{pmatrix}2\\5\end{pmatrix}$$

$$A_1 = B_1+C_1$$
$$V_1 = M_1.A$$

P1

```
read B1: 2
read C1: 0
calculate 2+0: 2
write 2 -> A1
acquire start_step2: ok
write 1  -> step1_finished
step1_finished == 2: false
release start_step2: ok
start_step2: wait
start_step2: wait
```

$$A_2 = B_2+C_2$$
$$V_2 = M_2.A$$

P2

```
read B2: 0
read C2: 0
calculate 5+0: 5
write 5-> A2
acquire start_step2: ok
write 2-> step1_finished
step1_finished == 2: true
```

Conditions example: vector mathematics

```
step1_finished=2

start_step2 = Condition()
```

$$B=\begin{pmatrix}2\\0\end{pmatrix}\quad C=\begin{pmatrix}0\\5\end{pmatrix}\quad M=\begin{pmatrix}1 & 2\\1 & 2\end{pmatrix}\quad A=\begin{pmatrix}2\\5\end{pmatrix}$$

$$A_1 = B_1+C_1$$
$$V_1 = M_1.A$$

P1

```
read B1: 2
read C1: 0
calculate 2+0: 2
write 2 -> A1
acquire start_step2: ok
write 1  -> step1_finished
step1_finished == 2: false
release start_step2: ok
start_step2: wait
start_step2: wait
start_step_2: wait
```

$$A_2 = B_2+C_2$$
$$V_2 = M_2.A$$

P2

```
read B2: 0
read C2: 0
calculate 5+0: 5
write 5-> A2
acquire start_step2: ok
write 2-> step1_finished
step1_finished == 2: true
notifyAll start_step_2: ok
```

Conditions example: vector mathematics

```
step1_finished=2
start_step2 = Condition()
```

$$B=\begin{pmatrix}2\\0\end{pmatrix}\quad C=\begin{pmatrix}0\\5\end{pmatrix}\quad M=\begin{pmatrix}1 & 2\\1 & 2\end{pmatrix}\quad A=\begin{pmatrix}2\\5\end{pmatrix}$$

$$A_1 = B_1+C_1$$
$$V_1 = M_1.A$$

P1

```
read B1: 2
read C1: 0
calculate 2+0: 2
write 2 -> A1
acquire start_step2: ok
write 1  -> step1_finished
step1_finished == 2: false
release start_step2: ok
start_step2: wait
start_step2: wait
start_step_2: wait
read M1: (1 2)
```

$$A_2 = B_2+C_2$$
$$V_2 = M_2.A$$

P2

```
read B2: 0
read C2: 0
calculate 5+0: 5
write 5-> A2
acquire start_step2: ok
write 2-> step1_finished
step1_finished == 2: true
notifyAll start_step_2: ok
```

Conditions example: vector mathematics

```
step1_finished=2
start_step2 = Condition()
```

$$B=\begin{pmatrix}2\\0\end{pmatrix}\quad C=\begin{pmatrix}0\\5\end{pmatrix}\quad M=\begin{pmatrix}1 & 2\\1 & 2\end{pmatrix}\quad A=\begin{pmatrix}2\\5\end{pmatrix}$$

$$A_1 = B_1+C_1$$
$$V_1 = M_1.A$$

P1

```
read B1: 2
read C1: 0
calculate 2+0: 2
write 2 -> A1
acquire start_step2: ok
write 1  -> step1_finished
step1_finished == 2: false
release start_step2: ok
start_step2: wait
start_step2: wait
start_step_2: wait
read M1: (1 2)
read A:(2 5)
```

$$A_2 = B_2+C_2$$
$$V_2 = M_2.A$$

P2

```
read B2: 0
read C2: 0
calculate 5+0: 5
write 5-> A2
acquire start_step2: ok
write 2-> step1_finished
step1_finished == 2: true
notifyAll start_step_2: ok

read M2(1 2)
```

Conditions example: vector mathematics

```
step1_finished=2
start_step2 = Condition()
```

$$B=\begin{pmatrix}2\\0\end{pmatrix}\quad C=\begin{pmatrix}0\\5\end{pmatrix}\quad M=\begin{pmatrix}1 & 2\\1 & 2\end{pmatrix}\quad A=\begin{pmatrix}2\\5\end{pmatrix}$$

$$A_1 = B_1+C_1$$
$$V_1 = M_1.A$$

P1

```
read B1: 2
read C1: 0
calculate 2+0: 2
write 2 -> A1
acquire start_step2: ok
write 1  -> step1_finished
step1_finished == 2: false
release start_step2: ok
start_step2: wait
start_step2: wait
start_step_2: wait
read M1: (1 2)
read A:(2 5)
calculate (1 2). (2 5): 12
```

$$A_2 = B_2+C_2$$
$$V_2 = M_2.A$$

P2

```
read B2: 0
read C2: 0
calculate 5+0: 5
write 5-> A2
acquire start_step2: ok
write 2-> step1_finished
step1_finished == 2: true
notifyAll start_step_2: ok

read M2(1 2)
read A: (2 5)
```

Deadlock

A condition in which threads are stuck waiting for each other forever

Deadlock example

Deadlock example

```
>>> x_lock = Lock()
```

Deadlock example

```
>>> x_lock = Lock()
>>> y_lock = Lock()
```

Deadlock example

```
>>> x_lock = Lock()
>>> y_lock = Lock()
>>> x = 1
```

Deadlock example

```
>>> x_lock = Lock()
>>> y_lock = Lock()
>>> x = 1
>>> y = 0
```

Deadlock example

```
>>> x_lock = Lock()
>>> y_lock = Lock()
>>> x = 1
>>> y = 0
>>> def compute():
```

Deadlock example

```
>>> x_lock = Lock()
>>> y_lock = Lock()
>>> x = 1
>>> y = 0
>>> def compute():
        x_lock.acquire()
```

Deadlock example

```
>>> x_lock = Lock()
>>> y_lock = Lock()
>>> x = 1
>>> y = 0
>>> def compute():
        x_lock.acquire()
        y_lock.acquire()
```

Deadlock example

```
>>> x_lock = Lock()
>>> y_lock = Lock()
>>> x = 1
>>> y = 0
>>> def compute():
        x_lock.acquire()
        y_lock.acquire()
        y = x + y
```

Deadlock example

```
>>> x_lock = Lock()
>>> y_lock = Lock()
>>> x = 1
>>> y = 0
>>> def compute():
        x_lock.acquire()
        y_lock.acquire()
        y = x + y
        x = x * x
```

Deadlock example

```
>>> x_lock = Lock()
>>> y_lock = Lock()
>>> x = 1
>>> y = 0
>>> def compute():
        x_lock.acquire()
        y_lock.acquire()
        y = x + y
        x = x * x
        y_lock.release()
```

Deadlock example

```
>>> x_lock = Lock()
>>> y_lock = Lock()
>>> x = 1
>>> y = 0
>>> def compute():
        x_lock.acquire()
        y_lock.acquire()
        y = x + y
        x = x * x
        y_lock.release()
        x_lock.release()
```

Deadlock example

```
>>> x_lock = Lock()
>>> y_lock = Lock()
>>> x = 1
>>> y = 0
>>> def compute():
        x_lock.acquire()
        y_lock.acquire()
        y = x + y
        x = x * x
        y_lock.release()
        x_lock.release()
>>> def anti_compute():
```

Deadlock example

```
>>> x_lock = Lock()
>>> y_lock = Lock()
>>> x = 1
>>> y = 0
>>> def compute():
        x_lock.acquire()
        y_lock.acquire()
        y = x + y
        x = x * x
        y_lock.release()
        x_lock.release()
>>> def anti_compute():
        y_lock.acquire()
```

Deadlock example

```
>>> x_lock = Lock()
>>> y_lock = Lock()
>>> x = 1
>>> y = 0
>>> def compute():
        x_lock.acquire()
        y_lock.acquire()
        y = x + y
        x = x * x
        y_lock.release()
        x_lock.release()
>>> def anti_compute():
        y_lock.acquire()
        x_lock.acquire()
```

Deadlock example

```
>>> x_lock = Lock()
>>> y_lock = Lock()
>>> x = 1
>>> y = 0
>>> def compute():
        x_lock.acquire()
        y_lock.acquire()
        y = x + y
        x = x * x
        y_lock.release()
        x_lock.release()
>>> def anti_compute():
        y_lock.acquire()
        x_lock.acquire()
        y = y - x
```

Deadlock example

```
>>> x_lock = Lock()
>>> y_lock = Lock()
>>> x = 1
>>> y = 0
>>> def compute():
        x_lock.acquire()
        y_lock.acquire()
        y = x + y
        x = x * x
        y_lock.release()
        x_lock.release()
>>> def anti_compute():
        y_lock.acquire()
        x_lock.acquire()
        y = y - x
        x = sqrt(x)
```

Deadlock example

```
>>> x_lock = Lock()
>>> y_lock = Lock()
>>> x = 1
>>> y = 0
>>> def compute():
        x_lock.acquire()
        y_lock.acquire()
        y = x + y
        x = x * x
        y_lock.release()
        x_lock.release()
>>> def anti_compute():
        y_lock.acquire()
        x_lock.acquire()
        y = y - x
        x = sqrt(x)
        x_lock.release()
```

Deadlock example

```
>>> x_lock = Lock()
>>> y_lock = Lock()
>>> x = 1
>>> y = 0
>>> def compute():
        x_lock.acquire()
        y_lock.acquire()
        y = x + y
        x = x * x
        y_lock.release()
        x_lock.release()
>>> def anti_compute():
        y_lock.acquire()
        x_lock.acquire()
        y = y - x
        x = sqrt(x)
        x_lock.release()
        y_lock.release()
```

Deadlock: example

```
def compute():
    x_lock.acquire()
    y_lock.acquire()
    y = x + y
    x = x * x
    y_lock.release()
    x_lock.release()
```

```
def anti_compute():
    y_lock.acquire()
    x_lock.acquire()
    y = y - x
    x = sqrt(x)
    x_lock.release()
    y_lock.release()
```

Deadlock: example

```
def compute():
      x_lock.acquire()
      y_lock.acquire()
      y = x + y
      x = x * x
      y_lock.release()
      x_lock.release()
```

```
def anti_compute():
      y_lock.acquire()
      x_lock.acquire()
      y = y - x
      x = sqrt(x)
      x_lock.release()
      y_lock.release()
```

```
compute()
```

```
anti_compute()
```

Deadlock: example

```
def compute():
    x_lock.acquire()
    y_lock.acquire()
    y = x + y
    x = x * x
    y_lock.release()
    x_lock.release()
```

```
def anti_compute():
    y_lock.acquire()
    x_lock.acquire()
    y = y - x
    x = sqrt(x)
    x_lock.release()
    y_lock.release()
```

Deadlock: example

```
def compute():
    x_lock.acquire()
    y_lock.acquire()
    y = x + y
    x = x * x
    y_lock.release()
    x_lock.release()
```

```
def anti_compute():
    y_lock.acquire()
    x_lock.acquire()
    y = y - x
    x = sqrt(x)
    x_lock.release()
    y_lock.release()
```

compute()	anti_compute()
P1 acquire x_lock: ok	P2 acquire y_lock: ok

Deadlock: example

```
def compute():
    x_lock.acquire()
    y_lock.acquire()
    y = x + y
    x = x * x
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def anti_compute():
    y_lock.acquire()
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    y = y - x
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    y_lock.release()
```

compute()	anti_compute()
P1 acquire x_lock: ok acquire y_lock: wait wait	P2 acquire y_lock: ok acquire x_lock:

Deadlock: example

```
def compute():
    x_lock.acquire()
    y_lock.acquire()
    y = x + y
    x = x * x
    y_lock.release()
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def anti_compute():
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    y = y - x
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    y_lock.release()
```

compute()	anti_compute()
P1	P2
acquire x_lock: ok	acquire y_lock: ok
acquire y_lock: wait	acquire x_lock:
wait	
wait	wait

Deadlock: example

```
def compute():
    x_lock.acquire()
    y_lock.acquire()
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compute()	anti_compute()
P1	P2
acquire x_lock: ok	acquire y_lock: ok
acquire y_lock: wait	acquire x_lock:
wait	
wait	wait
wait	wait

Deadlock: example

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compute()	anti_compute()
P1	P2
acquire x_lock: ok	acquire y_lock: ok
acquire y_lock: wait	acquire x_lock:
wait	
wait	wait
wait	wait
wait	wait

Deadlock: example

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def compute():
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    y_lock.acquire()
    x_lock.acquire()
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    y_lock.release()
```
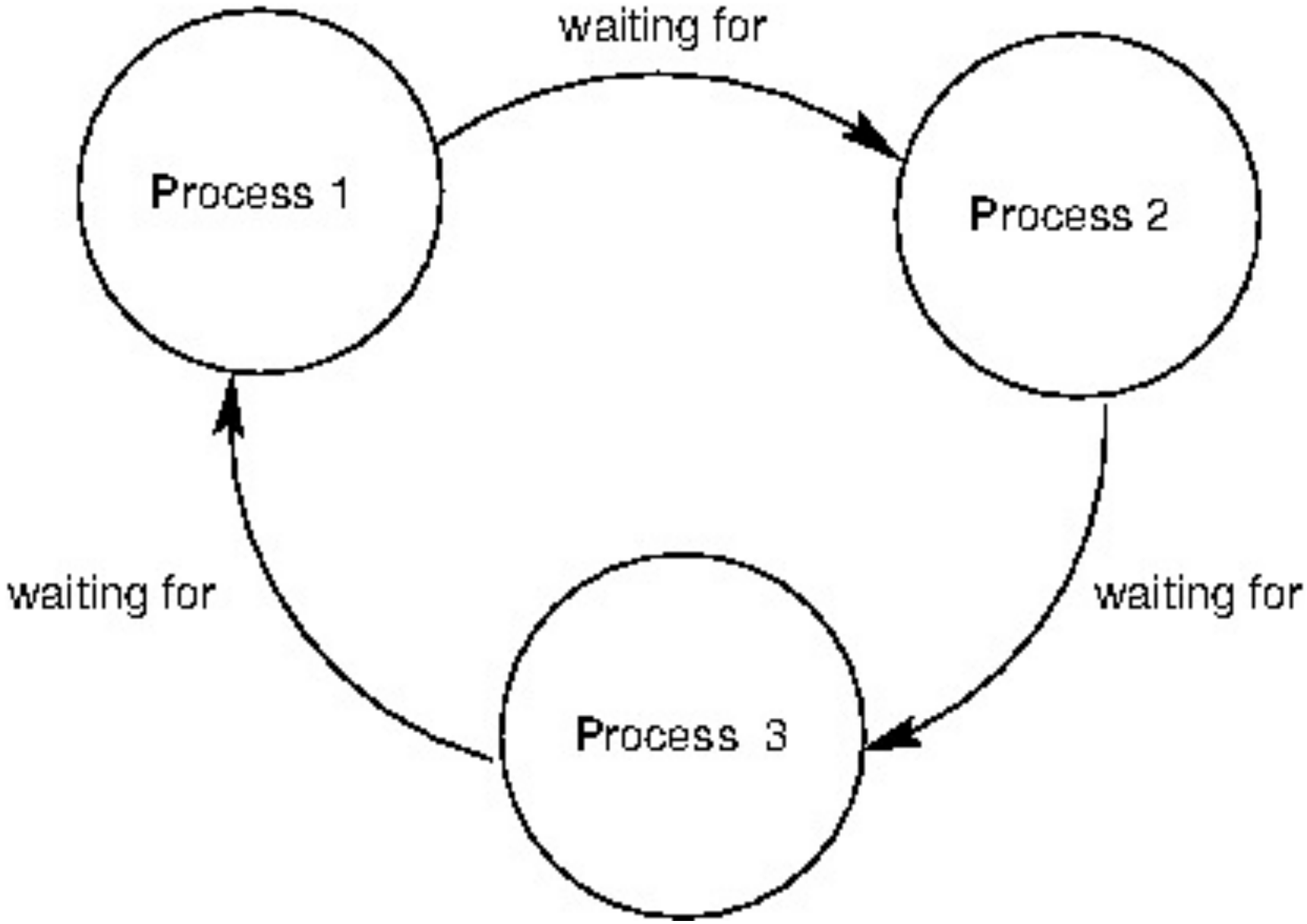

compute()	anti_compute()
P1	P2
acquire x_lock: ok	acquire y_lock: ok
acquire y_lock: wait	acquire x_lock:
wait	
wait	wait
wait	wait
wait	wait
...	...

Deadlock


waiting for
Process 1
Process 2
waiting for
waiting for
Process 3

Next time

Sequences and Streams