61A Lecture 18

Monday, October 10

Generic Functions, Continued

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A function might want to operate on multiple data types

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Last time:

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- Polymorphic functions using message passing

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What's different? Today's generic functions apply to multiple arguments that don't share a common interface

The Independence of Data Types

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add_rat mul_rat

Rational numbers as numerators & denominators

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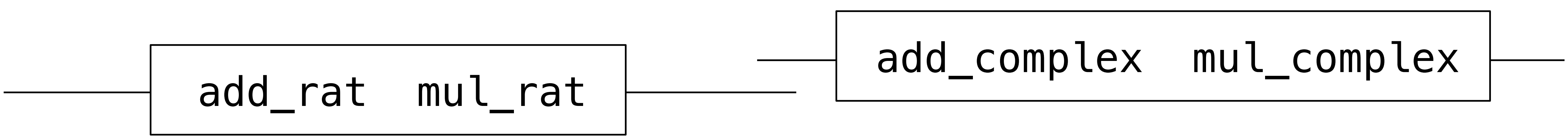

Rational numbers as numerators & denominators

Complex numbers as two-dimensional vectors

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How do we add a complex number and a rational number together?

add_rat mul_rat

Rational numbers as numerators & denominators

add_complex mul_complex

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How do we add a complex number and a rational number together?

`add_rat  mul_rat`

Rational numbers as numerators & denominators

`add_complex  mul_complex`

Complex numbers as two-dimensional vectors

There are many different techniques for doing this!

Rational Numbers, Now with Classes

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    def __init__(self, numer, denom):
        g = gcd(numer, denom)
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        return 'Rational({0}, {1})'.format(self.numer, self.denom)
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def add_rational(x, y):
    nx, dx = x.numer, x.denom
    ny, dy = y.numer, y.denom
    return Rational(nx * dy + ny * dx, dx * dy)
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Now with property methods, these might call functions

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Demo

Complex Numbers: the Rectangular Representation

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```
class ComplexRI(object):

    def __init__(self, real, imag):
        self.real = real
        self.imag = imag

    @property
    def magnitude(self):
        return (self.real ** 2 + self.imag ** 2) ** 0.5

    @property
    def angle(self):
        return atan2(self.imag, self.real)

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def add_complex(z1, z2):
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Might be either ComplexMA or ComplexRI instances

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Define a different function for each possible combination of types for which an operation (e.g., addition) is valid

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def iscomplex(z):
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def add_complex_and_rational(z, r):
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type_tag.tags = {ComplexRI: 'com',
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add.implementations = {}
add.implementations[('com', 'com')] = add_complex
add.implementations[('rat', 'rat')] = add_rational
add.implementations[('com', 'rat')] = add_complex_and_rational
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```
lambda r, z: add_complex_and_rational(z, r)
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add, subtract, multiply, divide

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Rational	Rational		
Complex	Rational		
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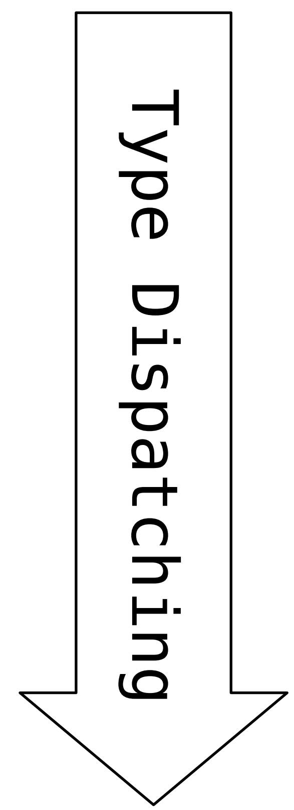

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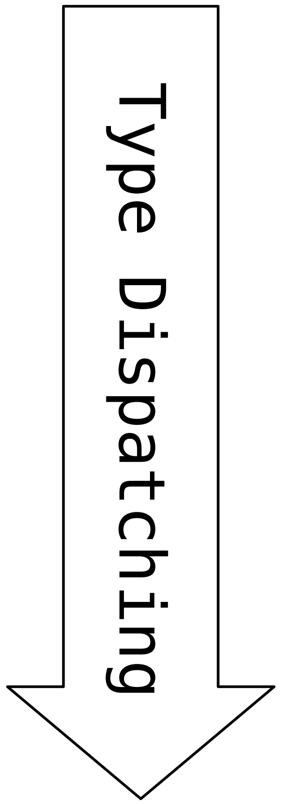

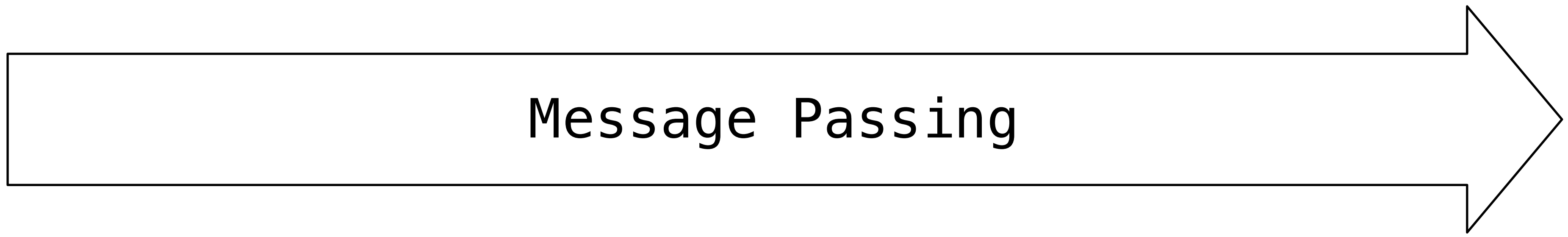

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Idea: One dispatch function for (operator, types) pairs

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```
def apply(operator_name, x, y):
    tags = (type_tag(x), type_tag(y))
    key = (operator_name, tags)
    return apply.implementations[key](x, y)
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Data-Directed Programming

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>>> def rational_to_complex(x):
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Takes advantage of structure in the type system

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>>> coercions = {('rat', 'com'): rational_to_complex}
```

Question: Can any numeric type be coerced into any other?

Coercion

Idea: Some types can be converted into other types

Takes advantage of structure in the type system

```
>>> def rational_to_complex(x):
        return ComplexRI(x.numer/x.denom, 0)

>>> coercions = {('rat', 'com'): rational_to_complex}
```

Question: Can any numeric type be coerced into any other?

Question: Have we been repeating ourselves with data-directed programming?

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Demo

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Type	Add	Multiply
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Rational		