61A Lecture 23

Friday, October 21

Sets

One more built-in Python container type

- Set literals are enclosed in braces
- Duplicate elements are removed on construction
- Sets are unordered, just like dictionary entries

```
>>> s = {3, 2, 1, 4, 4}
>>> s
{1, 2, 3, 4}
```

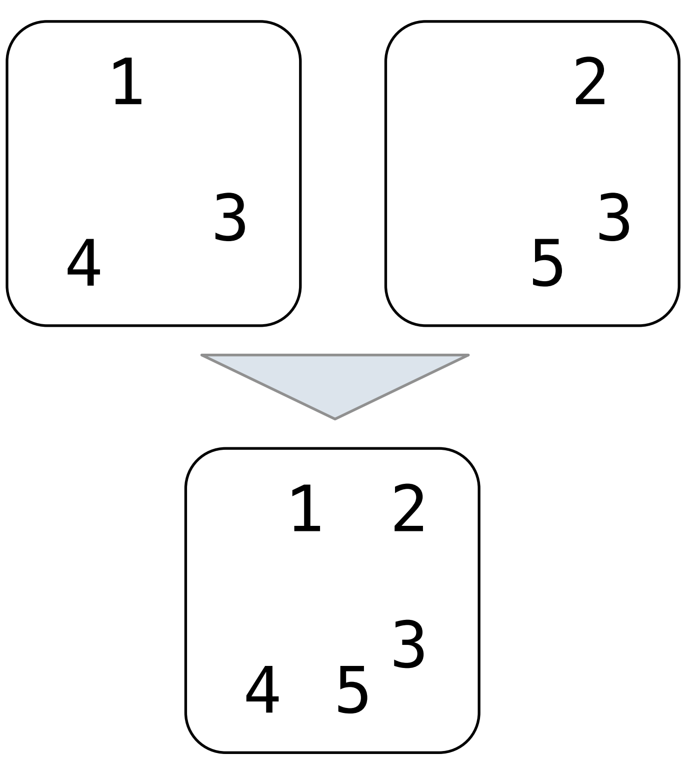
```
>>> 3 in s
True
>>> len(s)
4
>>> s.union({1, 5})
{1, 2, 3, 4, 5}
>>> s.intersection({6, 5, 4, 3})
{3, 4}
```

Implementing Sets

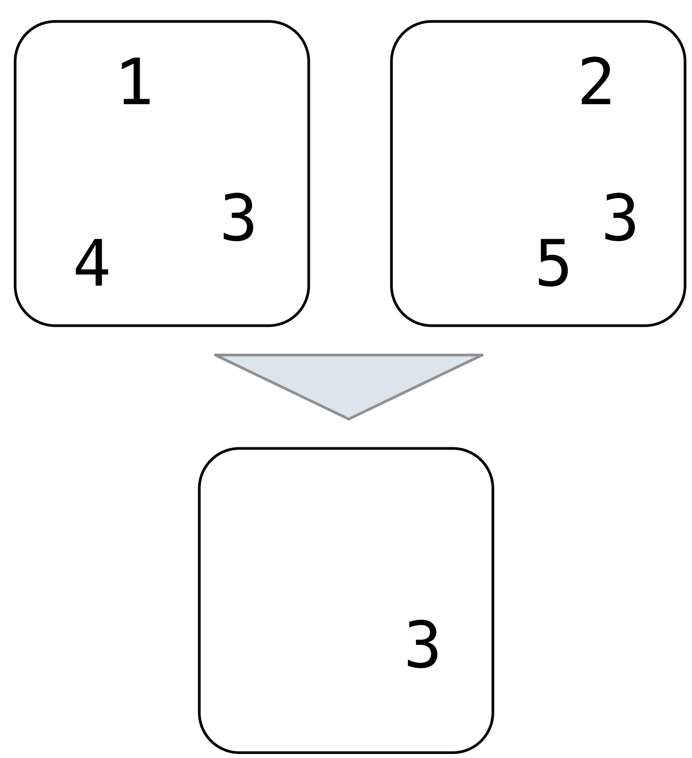
The interface for sets

- Membership testing: Is a value an element of a set?
- Union: Return a set with all elements in set1 **or** set2
- Intersection: Return a set with any elements in set1 **and** set2
- Adjunction: Return a set with all elements in s and a value v

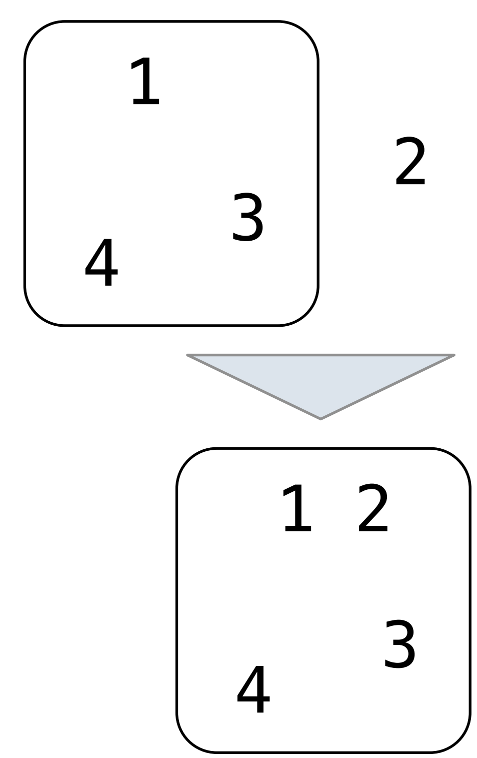
Union



Intersection



Adjunction



Sets as Unordered Sequences

Proposal 1: A set is represented by a recursive list that contains no duplicate items

```
def empty(s):
    return s is Rlist.empty

def set_contains(s, v):
    if empty(s):
        return False
    elif s.first == v:
        return True
    return set_contains(s.rest, v)
```

Demo

Review: Order of Growth

For a set operation that takes "*linear*" time, we say that

n: size of the set

R(n): number of steps required to perform the operation

$$R(n) = \Theta(n)$$

which means that there are constants k_1 and k_2 such that

$$k_1 \cdot n \leq R(n) \leq k_2 \cdot n$$

for sufficiently large values of ***n***.

Demo

Sets as Unordered Sequences

Time order of growth

```
def adjoin_set(s, v):
    if set_contains(s, v):
        return s
    return Rlist(v, s)
```

$\Theta(n)$

The size of the set

```
def intersect_set(set1, set2):
    f = lambda v: set_contains(set2, v)
    return filter_rlist(set1, f)
```

$\Theta(n^2)$

The size of the larger set

```
def union_set(set1, set2):
    f = lambda v: not set_contains(set2, v)
    set1_not_set2 = filter_rlist(set1, f)
    return extend_rlist(set1_not_set2, set2)
```

$\Theta(n^2)$

Sets as Ordered Sequences

Proposal 2: A set is represented by a recursive list with unique elements ordered from least to greatest

```
def set_contains2(s, v):
    if empty(s) or s.first > v:
        return False
    elif s.first == v:
        return True
    return set_contains2(s.rest, v)
```

Order of growth? $\Theta(n)$

Set Intersection Using Ordered Sequences

This algorithm *assumes* that elements are in order.

```
def intersect_set2(set1, set2):
        if empty(set1) or empty(set2):
            return Rlist.empty
        e1, e2 = set1.first, set2.first
        if e1 == e2:
            rest = intersect_set2(set1.rest, set2.rest)
            return Rlist(e1, rest)
        elif e1 < e2:
            return intersect_set2(set1.rest, set2)
        elif e2 < e1:
            return intersect_set2(set1, set2.rest)
```

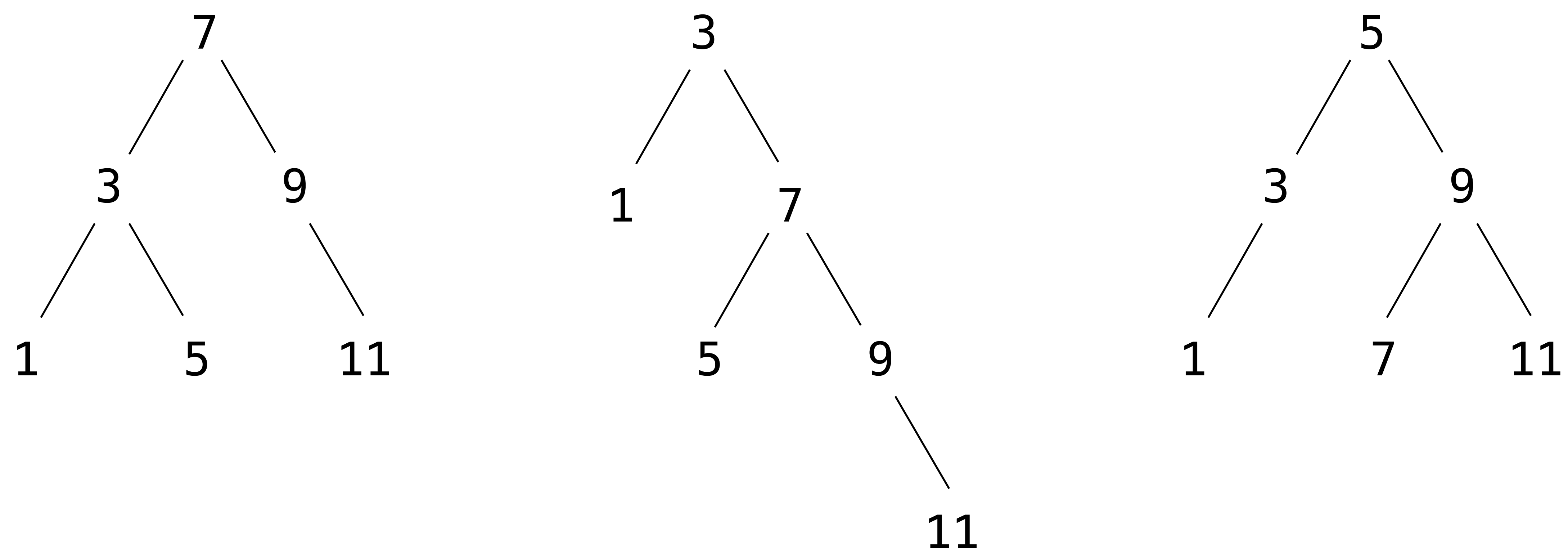
Demo

Order of growth? $\Theta(n)$

Tree Sets

Proposal 3: A set is represented as a Tree. Each entry is:

- Larger than all entries in its left branch and
- Smaller than all entries in its right branch

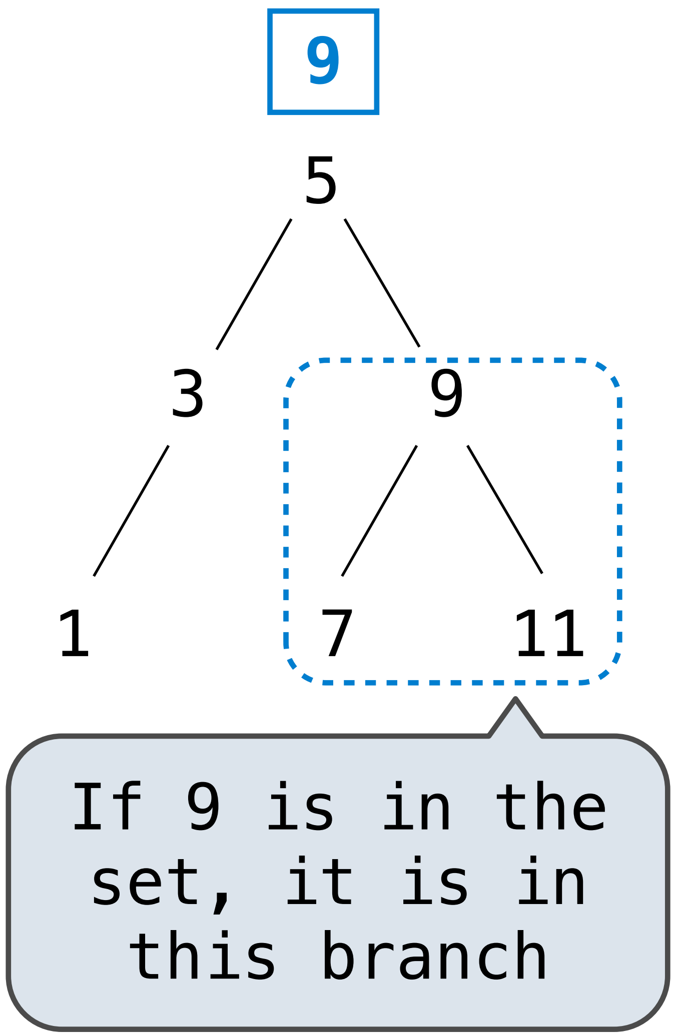


Membership in Tree Sets

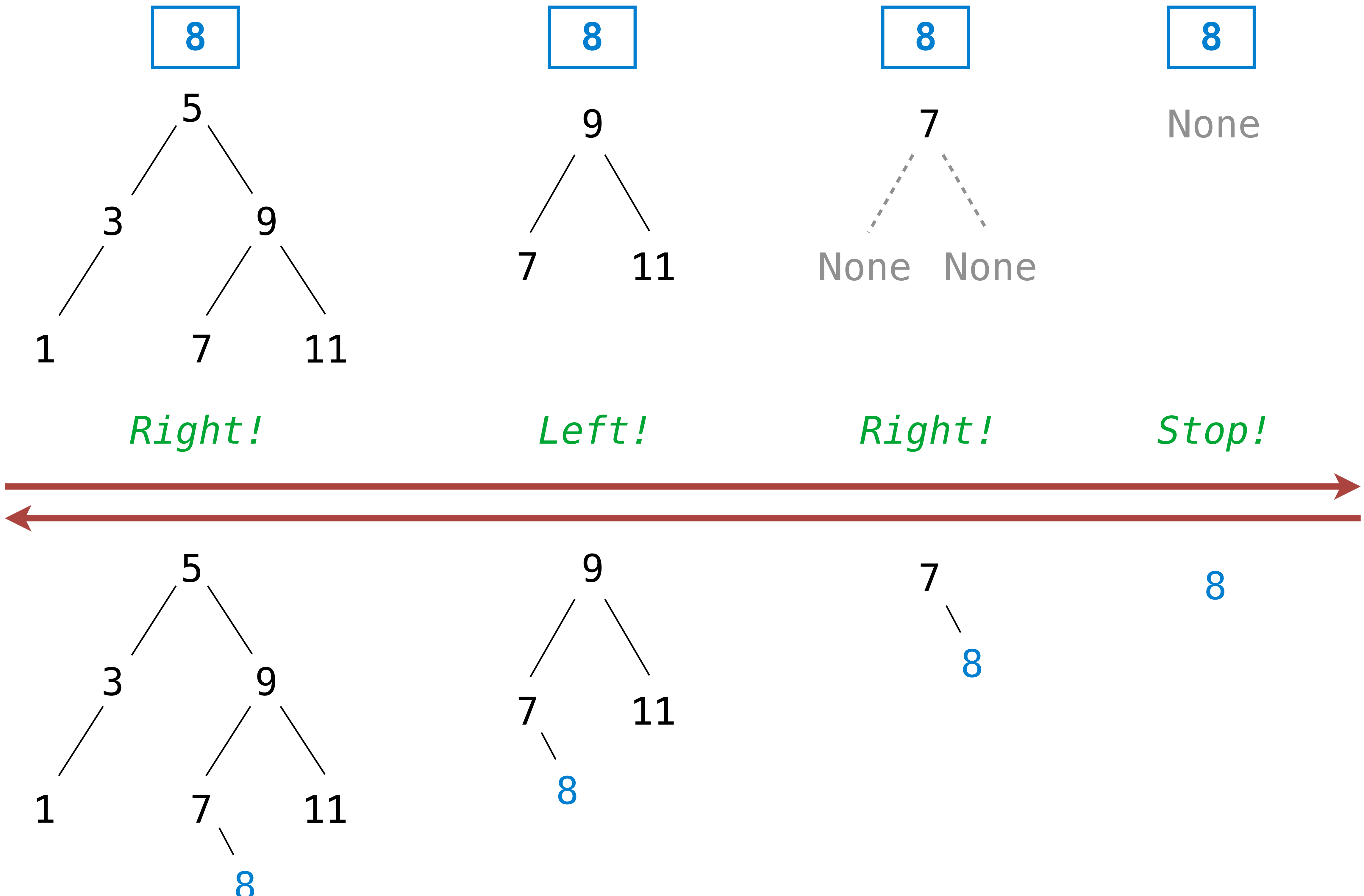
Set membership tests traverse the tree

- The element is either in the left or right sub-branch
- By focusing on one branch, we reduce the set by about half

```
def set_contains3(s, v):
    if s is None:
        return False
    elif s.entry == v:
        return True
    elif s.entry < v:
        return set_contains3(s.right, v)
    elif s.entry > v:
        return set_contains3(s.left, v)
```



Adjoining to a Tree Set



Demo

What Did I Leave Out?

Sets as ordered sequences:

- Adjoining an element to a set
- Union of two sets

Sets as binary trees:

- Intersection of two sets
- Union of two sets

That's homework 8!

No lecture on Monday

Midterm 2 on Monday, 7pm–9pm

Good luck!