1. Ohm's Law with noise

Sometimes we are quite fortunate to get nice numbers. Often times our measurement tools are a little bit noisy and values we get out of them are not accurate. However, if the noise is completely random then the effect of it can be averaged out over many samples. Say that we repeat our test on a different black box and now get the values

Test	i_{test} (mA)	v_{test} (V)
1	10	21
2	3	7
3	-1	-2
4	5	8
5	-8	-15
6	-5	-11

(a) Plot the measured voltage as a function of the current.

(b) Again we stack the currents and voltages to get $\vec{I} = \begin{bmatrix} 10 \\ 3 \\ -1 \\ 5 \\ -8 \\ -5 \end{bmatrix}$ and $\vec{V} = \begin{bmatrix} 21 \\ 7 \\ -2 \\ 8 \\ -15 \\ -11 \end{bmatrix}$. Can you solve for R this time? What conditions must $\vec{I}$ and $\vec{V}$ satisfy in order for us to solve for R? (Hint: Think about the range space of $\vec{I}$)

(c) Ideally, we would like to find R such that $\vec{V} = \vec{I}R$. If we cannot do this, we'd like to find a value of R that is the *best* solution possible, in the sense that $\vec{I}R$ is as "close" to $\vec{V}$ as possible. The idea of a best solution is subjective and dependent on the cost function we are using. One way of expressing this cost function in terms of R is to quantify the difference between each component of $\vec{V}$ (V_j) and each component of $\vec{I}R$ (I_jR), and add these "differences" up as follows:

$$cost(R) = \sum_{j=1}^{6} (V_j - I_jR)^2 \tag{1}$$

Do you think this is a good cost function? Why/why not?

(d) Show that you can also express the above cost function in vector form, that is,

$$cost(R) = \left\langle (\vec{V} - \vec{I}R), (\vec{V} - \vec{I}R) \right\rangle \tag{2}$$

(e) Find $\hat{R}$, the optimal R that minimizes $cost(R)$. Hint: Use calculus and minimize the expression in part c)!

(f) On your original IV plot, also plot the line $v = \hat{R}i$. Can you visually see why this line "fits" the data well? What if we had guessed $R = 3$? How well would we have done? What about $R = 1$?
Calculate the cost functions for each of these choices of R to validate your answer.

(g) Now, suppose we added a new data point: $i_7 = 2mA$, $v_7 = 4V$. Will $\hat{R}$ increase, decrease or remain the same? Why? What does that say about the line $v = \hat{R}i$?

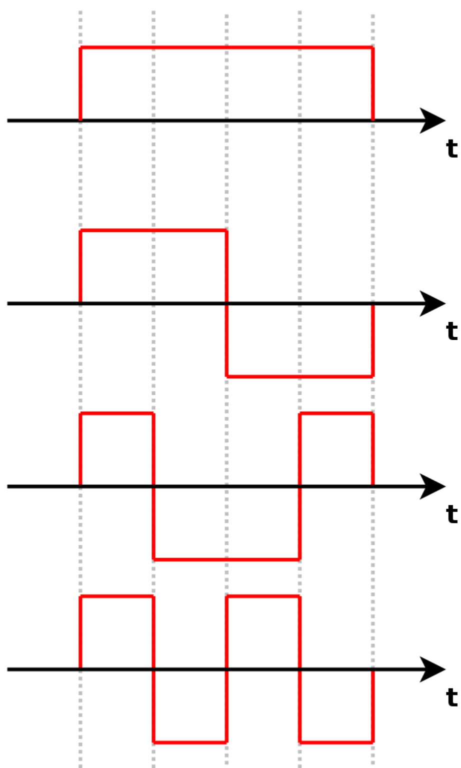
(h) Let's add another data point: $i_8 = 4mA, v_8 = 11V$. Will $\hat{R}$ increase, decrease or remain the same? Why? What does that say about the line $v = \hat{R}i$?

(i) Now, your mischievous friend has hidden the black box. You want to know what the output voltage would be across the terminals if you applied $5.5mA$ through the black box. What would your best guess be? (This is an example of estimation from machine learning! You have *learned* what is going on inside the black box by making observations; then used what you learned to make estimations.)

2. CDMA - Code division multiple access

Code division multiple access (CDMA) is a channel access method used by various radio communication technologies. CDMA is an example of multiple access, which is where several transmitters can send information simultaneously over a single communication channel. This allows several users to share a band of frequencies. To permit this without undue interference between the users, CDMA employs "spread-spectrum" technology and a special coding scheme (where each transmitter is assigned a code). These codes have special properties:

- Their auto-correlation with offset zero is very high and any other offset is very low.
- The cross-correlation between different codes is very low.



(a) Show that the "codes" represented by these signals are orthogonal.

(b) Sketch the auto-correlation of each code.

(c) Sketch the cross-correlation between any two pairs of codes.

(d) We now consider two senders and a single receiver. The senders want to talk at the same time.

- Sender 1 uses $\mathbf{v}_1 = \begin{bmatrix} 1 & -1 \end{bmatrix}$ to represent a 1 and $-\mathbf{v}_1 = \begin{bmatrix} -1 & 1 \end{bmatrix}$ to represent a 0.
- Sender 2 uses $\mathbf{v}_2 = \begin{bmatrix} 1 & 1 \end{bmatrix}$ to represent a 1 and $-\mathbf{v}_2 = \begin{bmatrix} -1 & -1 \end{bmatrix}$ to represent a 0.

If sender 1 has data $\begin{bmatrix} 1 & 0 & 1 & 0 \end{bmatrix}$ and sender 2 has data $\begin{bmatrix} 0 & 1 & 1 & 1 \end{bmatrix}$ to send, sketch the encoding process. What would be the raw signal received at the receiver (the signals add up since they were sent at the same time).

(e) If the above coding scheme was used by the senders (and they talked at the same time) and the received signal was $\begin{bmatrix} 0 & -2 & -2 & 0 & 2 & 0 & 2 & 0 \end{bmatrix}$, what were the two senders trying to send?