1. **Review**

2. **Mechanical Problems**

   In each part, find the eigenspace of M associated with the eigenvalue λ.

   (a) $M = \begin{bmatrix} 1 & 0 \\ 0 & 9 \end{bmatrix}, \lambda = 1.$

   (b) $M = \begin{bmatrix} 1 & 0 \\ 0 & 9 \end{bmatrix}, \lambda = 9.$

   (c) $M = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}, \lambda = 3.$

3. **Decomposition of Symmetric Matrices**

   Consider the matrix A below:

$$\begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & 2 \\ 2 & 2 & -1 \end{bmatrix}$$

   (a) Given that the eigenvalues for A are $\lambda_1 = 3$, $\lambda_2 = -3$, find the eigenvectors.

   (b) Notice that the eigenvectors between distinct eigenvalues are orthogonal! Now, prove that the eigenvectors between distinct eigenvalues of a symmetric matrix will *always* be orthogonal.

   (c) One of the applications where this technique (decomposing into an orthonormal eigenbasis) was useful was the Image Compression problem from the HW. What are some other applications where this could be useful?

4. Steady State Reservoir Levels We have 3 reservoirs, A, B and C. The pumps system between the reservoirs is depicted in Figure **??**.

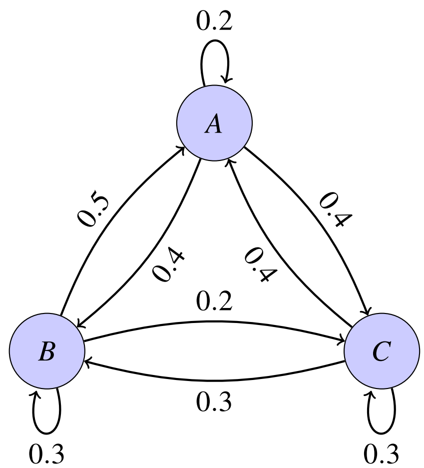

Figure 1: Reservoir pumps system

(a) Write the transition matrix representing the pumps system in the problem.

(b) Assuming you start the pumps with water levels $A_0 = 129, B_0 = 109, C_0 = 0$ (in kiloliters). What would be the steady state water levels (in kiloliters) according to the pumps system described in the problem?

Hint: If $\vec{x_{ss}} = \begin{bmatrix} A_{ss} \\ B_{ss} \\ C_{ss} \end{bmatrix}$ is a vector describing the steady state levels of water in the reservoirs (in kiloliters), what happens if you fill the reservoirs A, B and C with A_{ss}, B_{ss} and C_{ss} kiloliters of water, respectively and apply the pumps once?

Hint II: Note that the pumps system preserves the total amount of water in the reservoirs. That is, no water is lost or gained by applying the pumps.