



## 1. Circuits

Last week we saw how we could view circuits as traffic flows. Today we will go over how to incorporate node potentials into our model, so we can solve for the voltage across every element and the current through every element.

- Translate the above circuit into a directed graph, this time follow convention and label currents as going in through the + terminal and out through the - terminal. Write the incidence matrix for the graph. Ignore the current source for now.
- Let  $\vec{v}$  be the vector of node potentials. How many elements are in  $\vec{v}$ ? What is the product  $F\vec{v}$ ? What does this product represent?
- We now have a way to represent the voltage difference across elements. Last discussion we had defined a vector of currents through elements. What equation can be used to relate the voltage across and element and the current through it? Write down such an equation for each of the resistors in the above graph. Ignore the current source for now.
- Let  $R$  be the diagonal matrix of branch resistances. Write  $\vec{i}$  in terms of  $R$ ,  $F$ , and  $\vec{v}$ .
- The equation in d) represents the branch currents in terms of the node voltages. From last discussion we have another relation for the branch currents, namely,  $F^T\vec{i} = 0$ . Now we will model the current source as a steady current flowing into one node and a steady current flowing out of another. Let  $\vec{f}$  represent this vector of independent currents. Restate KCL to include this vector of independent sources  $\vec{f}$ .
- At this point we have two equations that independently describe the current through the branches ( $\vec{i}$ ). Use these equations to come up with one equation we could use to solve for the node potentials  $\vec{v}$ .
- Given we now have  $\vec{v}$ , how could we find  $\vec{i}$ ?