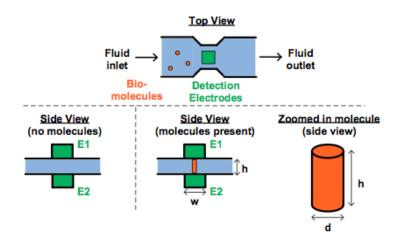
1. Bio-Molecule Detector

One application for electronics that has gained a lot of attention over the past several years is in so-called "bio-molecule" detection. The idea is to build a system that detects the presence of specific molecules and/or cells (e.g., specific viruses, proteins, etc.) in a biological sample; if this detection can be performed automatically and using relatively low-cost components, it can have a dramatic impact on a number of areas such as medical diagnosis, drug development, DNA sequencing, etc. In this problem we'll look at how some of the techniques we learned about in the touchscreen module can be applied to realize a hypothetical bio-molecule detector. (Real bio-molecule detection systems involve quite a bit more complexity than what we'll include here, but in many designs the same basic principles apply.)



As shown in Fig. 1 above, the detector works by flowing a liquid that may or may not contain the biomolecules through a region in the device that has electrodes on the top and bottom of the liquid channel. The electrodes (E1/E2 in Fig. 1) are chemically "functionalized" (using e.g. some appropriately designed antibodies) so that if the specific bio-molecule of interest is present in the fluid sample, one or more of the molecules will get physically trapped between the two electrodes (bottom right of Fig. 1). After all of the fluid has been cleared out of the device (i.e., so that if there are bio-molecules present, there is only air in between the two electrodes E1/E2), we can then figure out whether or not one or more bio-molecules were trapped by measuring the resistance between the two electrodes.

(a) Let's first assume that we want to detect the presence of a bio-molecule by measuring resistance. If no bio-molecule is present, what should be the resistance between E1/E2? As shown in Fig. 1, if each bio-molecule is a cylinder with diameter d = 10nm, height h = 100nm, and has a resistivity $\rho = 100 \ \mu\Omega^*$ m, what would be the resistance between E1 and E2 if only a single bio-molecule has been trapped? Note that you can assume that the trapped molecule is exactly vertically oriented when it is trapped - i.e., the top and bottom faces of the molecule are both aligned with surfaces of the electrodes.

(b)	Using the same numbers for d, h, and ρ as part a), as a function of the number of trapped bio-molecules $N_{molecules}$, what is the resistance between E1 and E2? (Note that you can assume that $N_{molecules}$ is small enough that all of the molecules fit within the electrode area, and that all of the molecules are still trapped in an exactly vertical orientation.)
(c)	Given your answers to parts (a) and (b), design a circuit that will output a voltage greater than 2.5V is more than 5 molecules are trapped.
(d)	Now let's assume that the circuit you designed in part (c) must be connected through a long cable to an oscilloscope (that measures the output voltage). If the resistance of this cable is $10k\Omega$ and the oscilloscope is an open circuit, will your design from part (c) still meet the required specifications?
(e)	Now let's assume that instead of being an open circuit, the oscilloscope can actually be modeled by a $100~k\Omega$ resistor (and that we still have the long cable connected between our design and the oscilloscope) - will your design from part (c) still meet the required specifications?
(f)	Knowing that you will always be operating under the scenario described in part (e), redesign your circuit (part c) so that it will definitely meet the required specifications.

2. Voltage Divider

Find the voltages across the resistors and the current source for the circuits below using KVL, KCL and Ohm's law.

