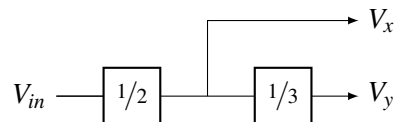## 1. Modular Circuits

In this problem we will explore the design of circuits that perform a set of (arbitrary) mathematical operations in order to elucidate some of the important properties and uses of op-amps in negative feedback. (Note that so-called analog signal processing - where these kinds of mathematical operations are performed on continuously-valued voltages by analog circuits - is extremely common in real-world applications; without this capability essentially none of our e.g. radios or sensors would actually work.) Specifically, let's assume that we want to implement the block diagram shown below:
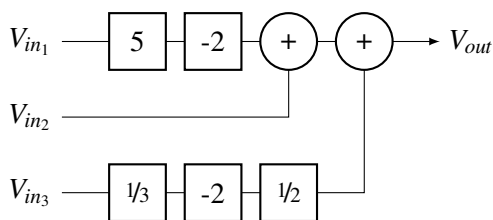

In other words, we want to implement a circuit with two outputs V_x and V_y, where $V_x = \frac{1}{2}V_{in}$, and $V_y = \frac{1}{3}V_x$.

(a) Design two voltage divider circuits that each independently would implement the two multiplications shown in the block diagram above (i.e., multiply by $1/2$, and multiply by $1/3$). Note that you do not need to include the input voltage sources in your design – you can simply define the input to each block as being at the appropriate potential (e.g., V_{in} or V_x).

(b) Assuming that V_{in} is created by an ideal voltage source, implement the original block diagram as a circuit by directly replacing each block with the designs you came up with in part (a).

(c) For the circuit from part (b), do you get the desired relationship between V_y and V_x? How about between V_x and V_{in}? Be sure to explain why or why not each block retains its desired functionality.

(d) Using an ideal op-amp in negative feedback, modify the design of one of the two blocks (i.e. the $1/2$ block or the $1/3$ block) so that the originally intended relationships between V_x and V_{in} as well as V_y and V_x are realized by the resulting overall circuit (where each block is replaced by its individual implementation). Is this configuration enough by itself to attach loads at V_x and V_y?

(e) Now let's assume that we want to expand our toolbox of circuits that implement mathematical operations. In particular, design blocks that implement:

   i. $V_o = 5V_i$
   ii. $V_o = -2V_i$
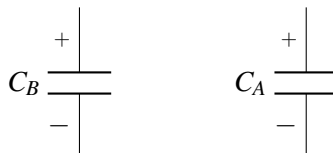   iii. $V_o = V_{i_1} + V_{i_2}$

   Pay careful attention to the way you design these blocks so that connecting any one block to any other block does not modify the intended functionality of any of the blocks.

(f) Check that your designs from part (e) indeed enable a library of compose-able elements (i.e. that you can connect any block to any other block without having the intended functionality be modified) by implementing the block diagram shown below.
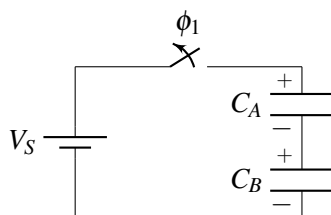
2. Capacitors and Charge Sharing Revisited

(a) Consider two capacitors, C_A and C_B that have been charged to Q_{A_1} and Q_{B_1} repectively. Compute the voltages across each one, V_{A_1} and V_{B_1}.

(b) Consider the following circuit set up with an open switch. Given that these were the same charged capacitors as in the previous part, in phase 1, what is the charge and voltage across both capacitors, **before** the switch is closed?

(c) Now, the switch is closed, and the circuit is allowed to settle. Compute V_{B_2}, the final voltage drop across C_B in terms of the given capacitances, the voltages computed in the first part, and V_S.