EECS 16A Designing Information Devices and Systems I
Fall 2016 Babak Ayazifar, Vladimir Stojanovic Homework 5

This homework is due October 4, 2016, at Noon.

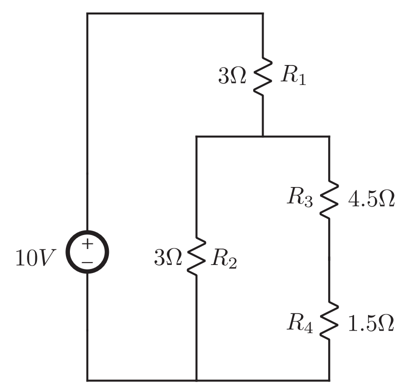
Optional Problems: We **do not** grade these problems. Nevertheless, you are responsible for learning the subject matter within their scope.
Bonus Problems: We **do** grade these problems. Doing them will provide an unspecified amount of extra credit; not doing them will not affect your homework grade negatively. We will specify if the problem is in or out of scope.

1. **Homework process and study group**

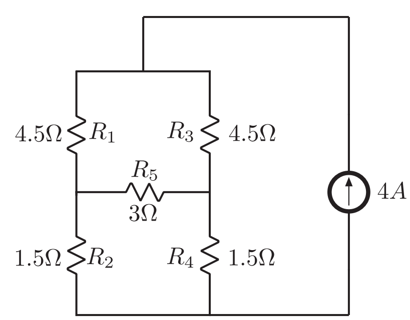
Who else did you work with on this homework? List names and student ID's. (In case of hw party, you can also just describe the group.) How did you work on this homework?
Working in groups of 3-5 will earn credit for your participation grade.

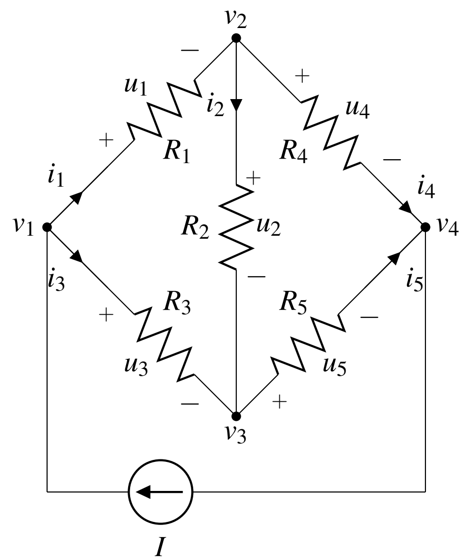
2. **Mechanical Circuits**

(a) Find the voltages across and currents flowing through all the resistors.



(b) Find the voltages across and currents flowing through all the resistors.





3. Circuits

In discussion 5A, we went over how to approach solving circuits from a linear algebraic perspective. We will now practice this technique with a slightly different circuit.

(a) Translate the above circuit into a directed graph, ignoring the current source for now. Write the incidence matrix for the graph.

(b) Let R be the diagonal matrix of branch resistances. Write Ohm's law as a matrix equation in terms of R, F, $\vec{i}$, and $\vec{v}$. Specify the contents of each vector/matrix that you use by writing out the individual elements.

(c) Let $\vec{f}$ represent the vector of independent currents, such that the KCL equation $F^T\vec{i} + f = 0$. Use this information, in addition to the previously derived equations to write $\vec{v}$ in terms of known quantities $(\vec{f}, G, F, R)$. You can use G as the conductance matrix.

(d) Now, use this information to write $\vec{i}$ in terms of known quantities $(\vec{f}, G, F, R, \vec{v})$.

(e) In an iPython notebook, solve for $\vec{v}$ and $\vec{i}$ in the given circuit. Let $R_1 = 100,000\Omega$, $R_2 = 200\Omega$ $R_3 = 100\Omega$ $R_4 = 100,000\Omega$ $R_5 = 100\Omega$ and $I = 3A$.

4. Cell Phone Battery

As great as smartphones are, one of the main gripes about them is that they need to be recharged too often. Suppose a Samsung Galaxy S3 requires about 0.4 W to maintain a signal as well as its regular activities (dominated by the display and backlight in many cases). The battery provides 2200 mAh at a voltage of 3.8V until it is completely discharged.

(a) How long will one full charge last you?

(b) Suppose the cell phone battery is completely discharged and you want to recharge it completely. How much energy (in J) is this? How much charge (in C) must be pumped through the battery?

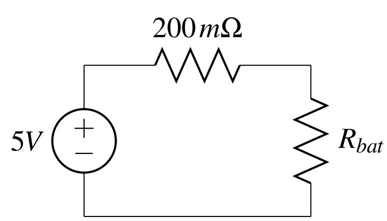
(c) Suppose PG&E charges $0.16 per kWh. Every day, you completely discharge the battery and recharge it at night. How much will recharging cost you for the month of October (31 days)?

(d) You are fed up with PG&E, gas companies, and Duracell/Energizer/etc. You want to generate your own energy and decide to buy a small solar cell (e.g. `http://ixdev.ixys.com/DataSheet/XOB17-Solar-Bit-Datasheet_Mar-2008.pdf`) for $1.50 on digikey. It delivers 40 mA at 0.5 V in bright sunlight. Unfortunately, now you have can only charge your phone when the sun is up. Using one solar cell, do you think there is enough time to charge a completely discharged phone every day? How many cells would you need to charge a completely discharged battery in an hour? How much will it cost you per joule if you have one solar cell that works for 10 years (assuming you can charge for 16 hours a day)? Do you think this is a good option?

(e) The battery has a lot of internal circuitry that prevents it from getting overcharged (and possibly exploding!) as well as transfering power into the chemical reactions used to store energy. We will model this internal circuitry as being one resistor with resistance R_{bat}, which you can set to any non-negative value you want. Furthermore, we'll assume that all the energy dissipated across R_{bat} goes to recharging the battery. Suppose the wall plug and wire can be modeled as a 5V voltage source and 200 mΩ resistor, as pictured in Fig. **??**. What is the power dissipated across R_{bat} for R_{bat}= 1mΩ , 1Ω, and 10kΩ? How long will the battery take to charge for each of those values of R_{bat}?

(f) (Bonus) Suppose you forgot to charge your phone overnight, and you're in a hurry to charge it before you leave home for the day. What should we set R_{bat} to be if we want to charge our battery as quickly as possible? How much current will this draw? How long will it take to charge?

Hint: what choice of R_{bat} maximizes the power dissipated across the resistor?



Model of wall plug, wire, and battery.

(g) (Bonus) You might have found that the answer for the previous section seemed to waste a lot of energy. If you don't forget to charge your phone overnight, you have all 8 hours that you spend sleeping to charge your phone. What should you choose for R_{bat} to minimize the amount of wasted energy, while still charging the battery in no longer than 8 hours? Compare the power dissipated across the wire and the power dissipated across R_{bat}. Use the same model from Fig. **??**

5. Temperature Sensor

Measuring quantities in the physical world is the job of sensors. This means somehow extracting that information from the world and then converting it into a form that can be observed and processed. Electric circuits can be very useful for doing this.

For most materials, resistance increases with increasing temperature; that is, a resistor has higher resistance when it is hot than when it is cold. This is often an annoyance to circuit designers who want their circuits to work the same way at different temperatures, but this fact can also be useful. It allows us to convert temperature, a "physical" quantity, into resistance, an "electrical" quantity, to build an electronic thermometer.

A PT100 is a special resistor made out of platinum that has a very precise relationship between resistance and temperature. At 0°C, the PT100 is a 100Ω resistor. Taking the data from `http://www.hayashidenko.co.jp/en/info12.html`, we found that the positive temperature coefficient for a PT100 is approximately $0.366\Omega/°C$, that is, an increase in temperature of 1°C increases the PT100's resistance by 0.366Ω.

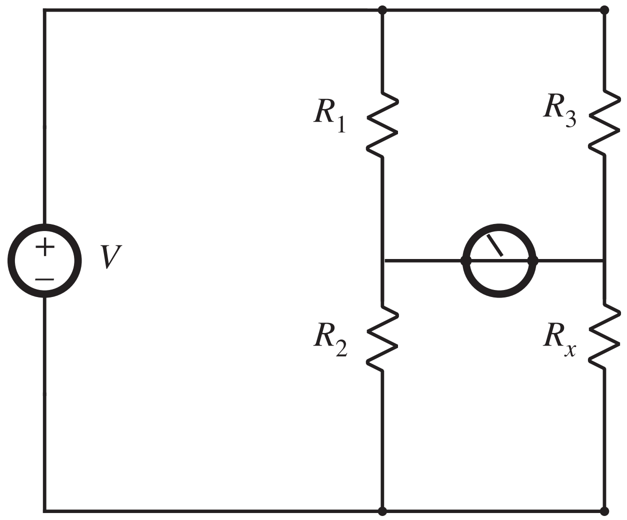


Figure 1: Circuit to measure resistance.

Consider the circuit in Fig. 1. It allows measuring resistance very precisely, as we will see below. The circle in the middle of the resistors is a *galvanometer*. It functions like an ideal wire [1] , but it also detects any current going through it.

(a) We say that the circuit is balanced when the current across the galvanometer in the middle is 0. Derive a relationship for the unknown resistance R_x in terms of the other three resistances if this is the case.

(b) We can thus find one resistance if we know the other three. Suppose $R_1 = 50\Omega$, $R_3 = 100\Omega$ and R_2 can be adjusted from 0 to 300Ω. This adjustment can be used to balance the circuit. What is the maximum resistance that can thus be measured for R_x? (Only using the fact that the circuit is balanced when R_2 is set appropriately).

(c) Assume R_x in fig. 1 is a PT100. Give a procedure by which you can find the temperature of the resistor. What is the maximum temperature you can measure, and why?

(d) Suppose the company manufacturing your resistors gave you some parts from a bad batch, and instead of being 100Ω, R_3 was actually some random number between 95 and 105Ω (i.e. $R_3 = (1+\varepsilon)100\Omega$ for $|\varepsilon| \leq 0.05$). Unfortunately, you didn't realize this and assumed it was still 100Ω. What is the biggest (in magnitude) error this will introduce to your temperature measurement?

(e) Now assume both R_1 and R_3 came from the same bad batch, so

$$R_1 = (1+\varepsilon)50\Omega$$
$$R_3 = (1+\varepsilon)100\Omega$$

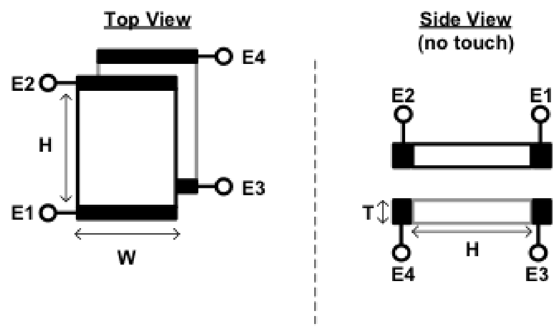
where both R_1 and R_3 have the same ε (still $|\varepsilon| \leq 0.05$). How much error will this introduce to the temperature measurement?

[1] In fact, galvanometers can be constructed as essentially just a coil of wire – current passing through the coil creates a magnetic field, which deflects the needle of a compass according to the strength and direction of the current. This is another wonderful property of electricity – it can be harnessed to have physical macroscale-level effects on the world that are observable by people.

(f) In the setup of the earlier parts (where $R_1 = 50\Omega$ and $R_3 = 100\Omega$ exactly), suppose we can only adjust R_2 in increments of 10Ω. Assume the galvanometer displays the direction of current flow (or 0 if no current). By adjusting R_2 in increments and observing the direction of current flow across the galvanometer, to what accuracy can we measure temperature?

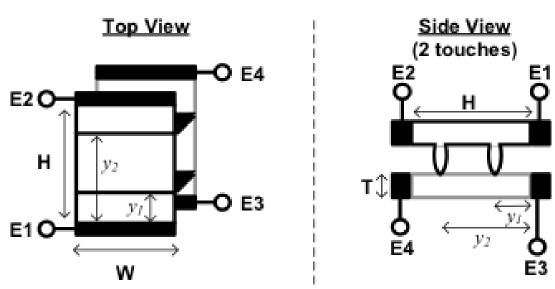
6. Multitouch Resistive Touchscreen

In this problem we will look at a simplified version of the multitouch resistive touchscreen. In particular, rather than measuring the position of two potential touch points in both dimensions (i.e., a pair of coordinates (x_1, y_1) and (x_2, y_2) corresponding to two touch positions), let's think about a version where we are interested in measuring only the vertical position of the two touch points (i.e., y_1 and y_2). Therefore, unlike the touchscreens we looked at in class and as shown below, both of the resistive plates (i.e., both the top and the bottom plate) would have conductive strips placed along their top and bottom edges.

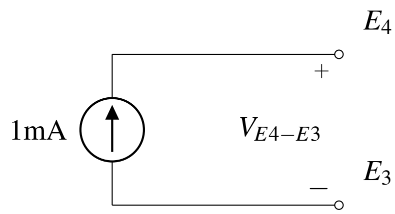


(a) Assuming that both of the plates are made out of a material with $\rho = 1\Omega m$ and that the dimensions of the plates are $W = 3cm$, $H = 12cm$, and $T = 0.5mm$, with no touches at all, what is the resistance between terminals E_1 and E_2 (which would be the same as the resistance between terminals E_3 and E_4)?

(b) Now let's look at what happens when we have two touch points. Let's assume that at wherever height the touch occurs, a perfect contact is made between the top plate and the bottom plate along the entire width of the plates (i.e., you don't have to worry about any lateral resistors), but that otherwise none of the electrical characteristics of the plates change. Defining the bottom of the plate as being $y = 0cm$ (i.e., a touch right at E_1 would be at $y = 0cm$), let's assume that the two touches happen at $y_1 = 3cm$ and $y_2 = 7cm$, and that your answer to part (a) was $5k\Omega$ (which may or may not be the right answer), draw a model with 6 resistors that captures the electrical connections between E_1, E_2, E_3, and E_4. Note that for clarity, the system has been redrawn below to depict this scenario.



(c) Using the same assumptions as part b), if you drove terminals E_3 and E_4 with a 1mA current source (as shown below) but left terminals E_1 and E_2 open-circuited, what is the voltage you would measure across $E_4 - E_3$ (i.e., V_{E4-E3})?



(d) Now let's try to generalize the situation by assuming that the two touches can happen at any two arbitrary points y_1 and y_2, but with y_1 defined to always be less than y_2 (i.e., y_1 is always the bottom touch point). Leaving the setup the same as in part c) except for the arbitrary y_1 and y_2, by measuring only the voltage between E_4 and E_3, what information can you extract about the two touch positions? Please be sure to provide an equation relating V_{E4-E3} to y_1 and y_2 as a part of your answer, and note that you may want to redraw the model from part (b) to help you with this.

(e) One of your colleagues claims that by measuring the appropriate voltages, not only can they extract what both y_1 and y_2 are in this system, they can even do so in a way that would have a set of three independent voltage equations related to y_1 and y_2. As we will see later, this will allow us to gain some robustness to noise in the voltage measurements.

In order to facilitate this, write equations relating V_{E4-E2} and V_{E1-E3} to y_1 and y_2. (The third voltage we'll use is V_{E4-E3}, which you should have already derived an equation for in the previous part of the problem.)

7. Your Own Problem Write your own problem related to this week's material and solve it. You may still work in groups to brainstorm problems, but each student should submit a unique problem. What is the problem? How to formulate it? How to solve it? What is the solution?