This homework is due October 11, 2016, at Noon.

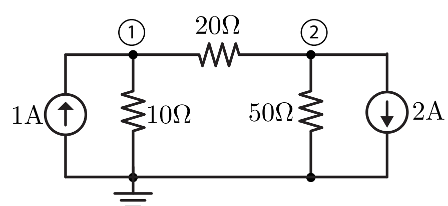
1. **Homework process and study group**

   Who else did you work with on this homework? List names and student ID's. (In case of hw party, you can also just describe the group.) How did you work on this homework?
   Working in groups of 3-5 will earn credit for your participation grade.

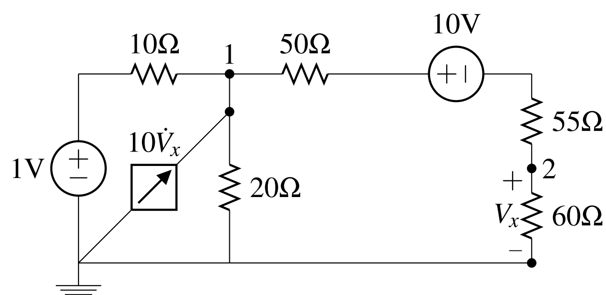
2. **Nodal Analysis**

   Using techniques presented in class, label all unknown node voltages and apply KCL to each node to find all the node voltages.

   (a) Solve for all node voltages using nodal analysis. Verify with superposition.

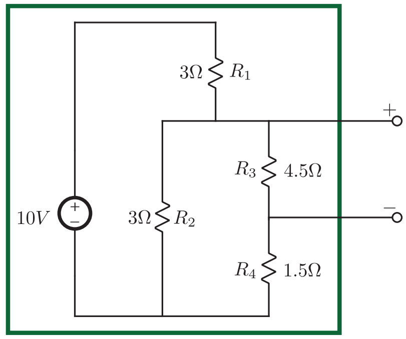


   (b) Solve for all node voltages using nodal analysis.

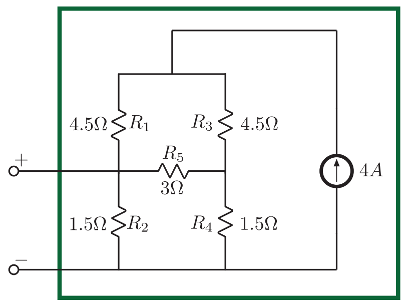


3. **Thévenin and Norton equivalent circuits**

   (a) Find the Thévenin and Norton equivalent circuits seen from the outside the box.

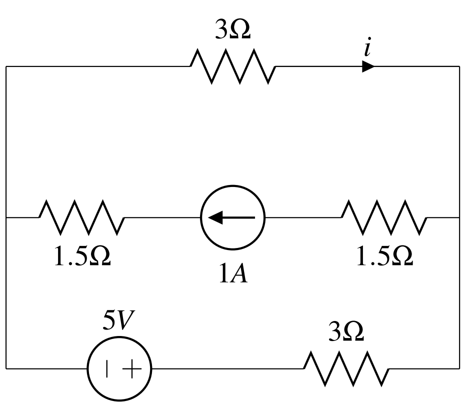


(b) Find the Thévenin and Norton equivalent circuits seen from the outside the box.



4. Nodal Analysis Or Superposition?

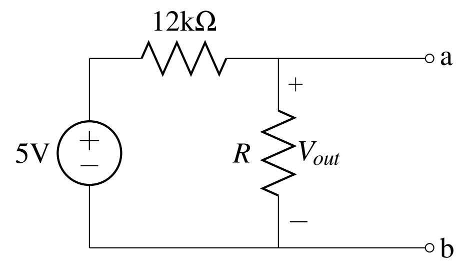
Solve for the current through the 3Ω resistor, marked as i, using superposition. Verify using nodal analysis. You can use IPython to solve the system of equations if you wish. Where did you place your ground, and why?



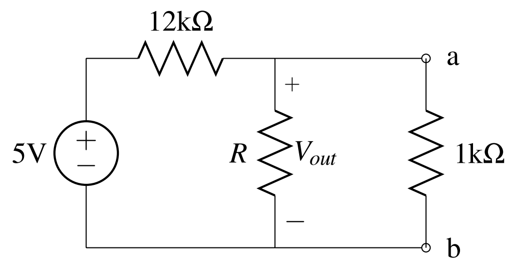
5. (OPTIONAL) Resistive Voltage “Regulator"

In this problem, we will design a circuit that provides an approximately constant voltage divider across a range of loads. We will use a resistor divider circuit as seen in discussion. The goal is to design a circuit that from a source voltage of 5V would yield an output voltage within 5% of 4V for loads in the range of $1\text{k}\Omega$ to $100\text{k}\Omega$.

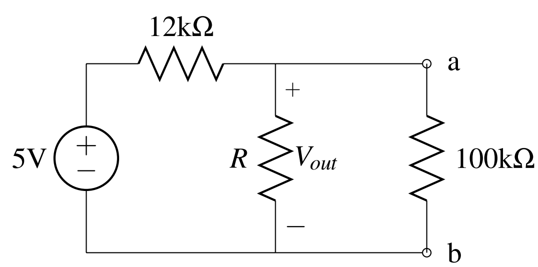
(a) First, consider the resistive voltage divider in the following circuit. What value of the resistor R would achieve a voltage V_{out} of 4V?



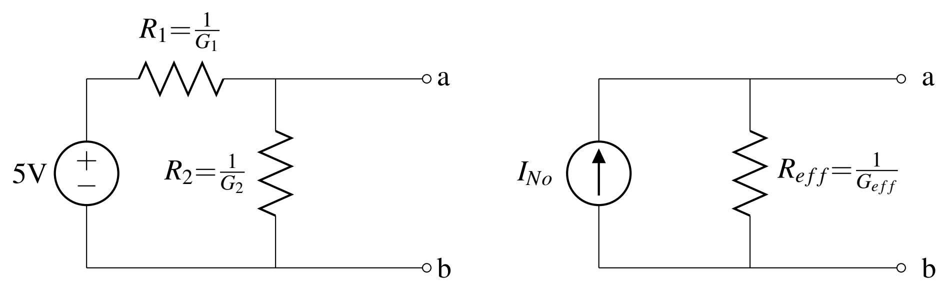
(b) Now consider loading the circuit with a resistor of 1kΩ as depicted in the following circuit with the same value of the resistor R as calculated in part (a). What is the voltage V_{out} now?



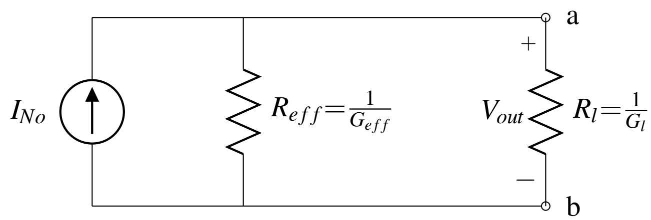
(c) Now consider loading the circuit with a resistor of 100kΩ, instead, as depicted in the following circuit with the same value of the resistor R as calculated in part (a). What is the voltage V_{out} now?



(d) Now we would like to design a divider that would keep the voltage V_{out} regulated for loads for a range of loads R_l. By that, we would like the voltage to remain within 5% window of 4V. That is, we would like to design the following circuit such that $3.80\text{V} \leq V_{out} \leq 4.20\text{V}$ for a range of loads R_l. As a first step, what is the Norton equivalent of the circuit on the left? Write I_{No} and G_{eff} it in terms of conductance values $G_1 = \frac{1}{R_1}$ and $G_2 = \frac{1}{R_2}$.



(e) The second step, using the Norton equivalent circuit you found in part (d), what is the range of G_{eff} that achieves $3.80\text{V} \leq V_{out} \leq 4.20\text{V}$ in terms of I_{No} and G_l?



(f) Translate the range of G_{eff} in terms of I_{No} and G_l (that you found in part (e)) into a range on G_2 in terms of G_1 and G_l.

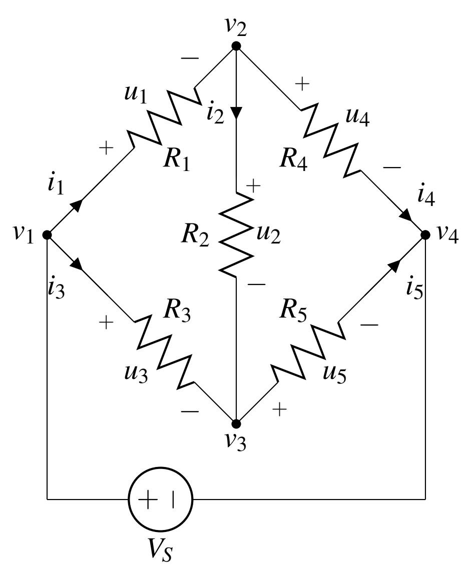
(g) Say we want to support loads in the range $1\text{k}\Omega \leq R_l \leq 100\text{k}\Omega$ with approximately constant voltage as described above (that is, $3.80\text{V} \leq V_{out} \leq 4.20\text{V}$). What is the range of G_2 in terms of G_1 now? Translate the range of G_2 in terms of G_1 into a range of R_2 in terms of R_1.

(h) Note that conductance is always non-negative. From the bounds on G_2 you found in the previous part, derive a bound on G_1 that ensures that G_2 is always non-negative and non-empty (that is, the whole range of possible G_2 values is non-negative and is not empty). Translate this range into a range of possible R_1 values. (Hint: In addition to the conductance being non-negative, also make sure that the range for G_2 is non-empty.)

(i) Pick the values of R_1 and R_2 that achieve $3.80\text{V} \leq V_{out} \leq 4.20\text{V}$ for $1\text{k}\Omega \leq R_l \leq 100\text{k}\Omega$ while minimizing the power consumed by the voltage divider circuit in open circuit (when there is no load attached to the output). What are these values R_1 and R_2? How much power is consumed in this case? Calculate and report this power consumption using both the original circuit and the Norton equivalent circuit. Are the power you calculated using the original circuit and the power you calculated using the Norton equivalent circuit equal?

(j) Now using the same values R_1 and R_2 from the previous part, load the circuit with load of $51\text{k}\Omega$, how much is consumed by each of the three resistors, R_1, R_2 and R_l (use the original circuit to compute the power)?

6. Solving Circuits with Voltage Sources



In the last homework, we implemented a circuit solver in an iPython notebook. This week we will make a small extension to allow us to solve circuits with voltage sources.

(a) What relationship does the voltage source enforce between v_1 and v_4?

(b) As you saw above, voltage sources will fix the nodes they are attached to be a constant offset from each other. We will treat v_1 and v_4 as one node, and our new vector of potentials will be $\vec{v} = \begin{bmatrix} v_2 \\ v_3 \\ v_4 \end{bmatrix}$. For the circuit above, draw the graph for the circuit where v_1 and v_4 are combined into one node. Specify a new incidence matrix for this graph.

(c) Previously we wrote Ohm's law as a matrix equation: $F\vec{v} = R\vec{i}$. Using our new incidence matrix, for every Ohm's law equation involving v_1, we will need to account for the constant offset by the voltage source. Find R and $\vec{b}$ so that Ohm's law is written $F\vec{v} + \vec{b} = R\vec{i}$.

(d) From before, we wrote KCL as $F^T\vec{i} + \vec{f} = 0$. (What is $\vec{f}$ in this circuit?) Use this information, in addition to the previously derived equation to write $\vec{v}$ in terms of known quantities ($\vec{f}, \vec{b}, G, F, R$). You can use G as the conductance matrix. You may need to modify several of the members of the derived equation by grounding a node and dropping a row or a column in order to give the problem a unique solution.

(e) Now, use this information to write $\vec{i}$ in terms of known quantities ($\vec{f}, G, F, R, \vec{v}$) and the quantities defined the the previous part (which should also be derived from known quantities).

(f) In an iPython notebook, solve for $\vec{v}$ and $\vec{i}$ in the given circuit. Let $R_1 = 100,000\Omega$, $R_2 = 200\Omega$ $R_3 = 100\Omega$ $R_4 = 100,000\Omega$ $R_5 = 100\Omega$ and $V_S = 10V$.