

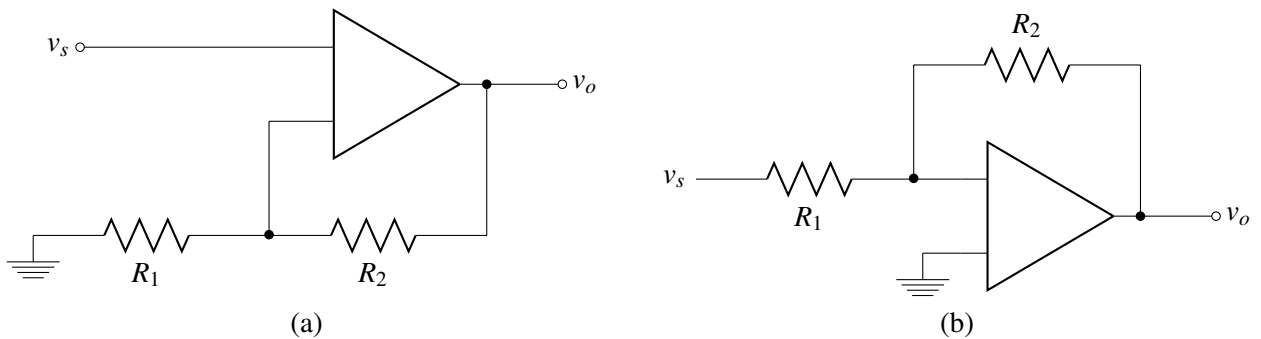
**This homework is due October 25, 2016, at 1PM.**

**1. Homework process and study group**

Who else did you work with on this homework? List names and student ID's. (In case of hw party, you can also just describe the group.) How did you work on this homework?  
Working in groups of 3-5 will earn credit for your participation grade.

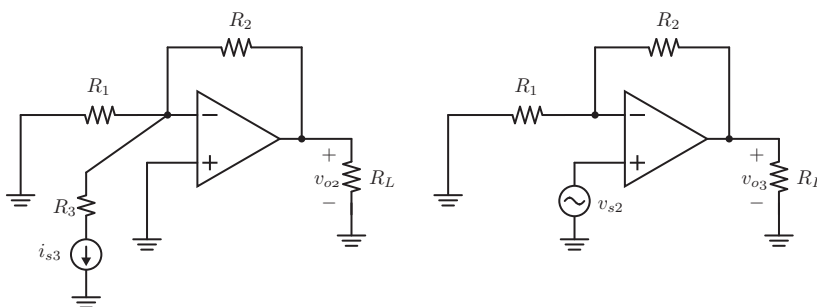
**2. Basic Amplifier Building Blocks**

The following amplifier stages are used often in many circuits and are well known as (a) the non-inverting amplifier and (b) the inverting amplifier.



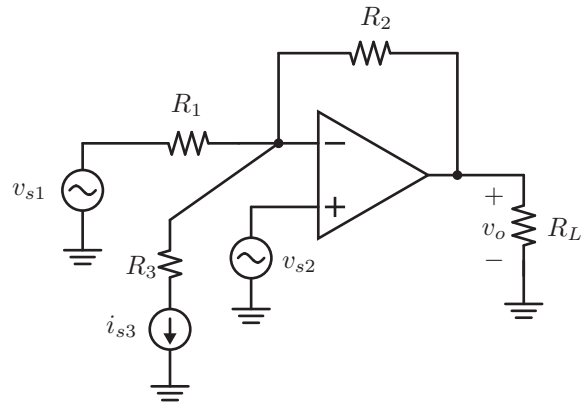
- (a) Label the input terminals of the Op-amp so it is in negative feedback. Then, derive the voltage gain of the non-inverting amplifier using the Golden Rules. Explain the origin of the name of the amplifier.
- (b) Label the input terminals of the Op-amp so it is negative feedback. Then, derive the voltage gain of the inverting amplifier using the Golden Rules. Explain the origin of the name of the amplifier.

**3. Amplifier with Multiple Inputs**

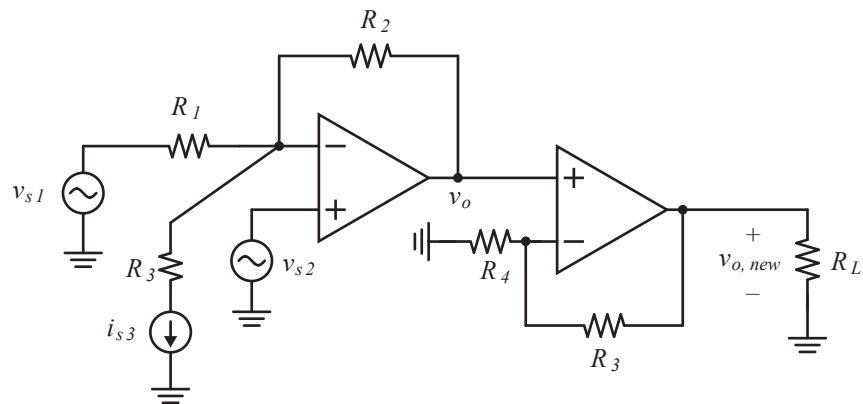


- (a) Use the Golden Rules to find  $v_{o2}$  for the first circuit above.
- (b) Use the Golden Rules to find  $v_{o3}$  for the second circuit above.

(c) Use the Golden Rules to find the output voltage  $v_o$  for the circuit shown below.

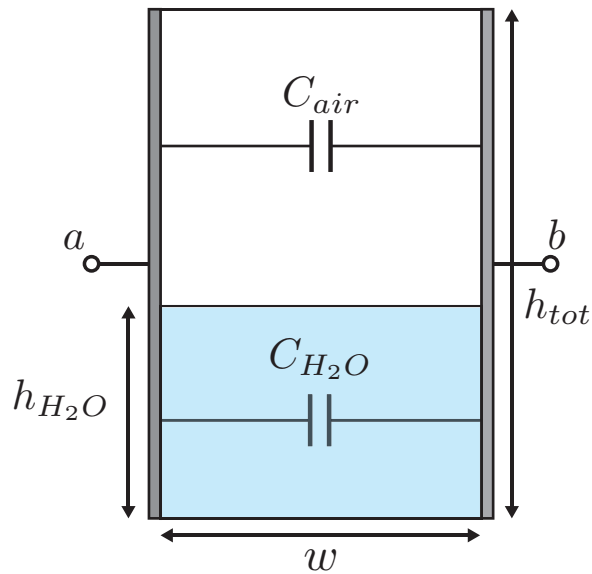


(d) Now add a second stage as shown below. What is  $v_{o, new}$ ? Does  $v_o$  change between the last part and this part? Does the voltage  $v_{o, new}$  depend on  $R_L$ ?



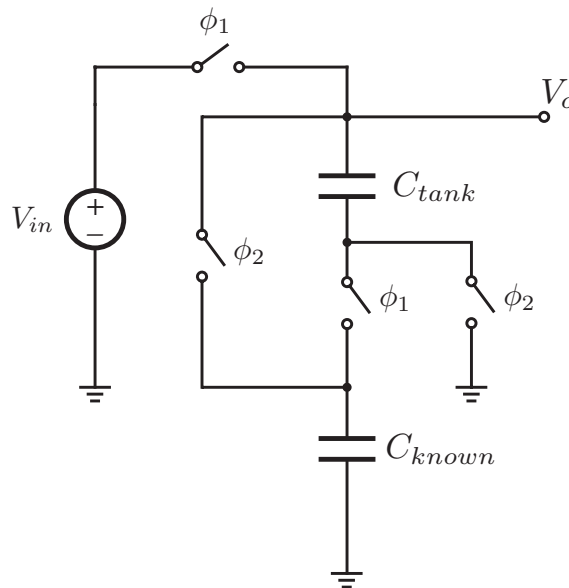
#### 4. It's finally raining!

A lettuce farmer in the Salinas valley has grown tired of weather.com's imprecise rain measurements. So, she decided to take matters into her own hands by building a rain sensor. She placed a rectangular tank outside and attached two metal plates to two opposite sides in an effort to make a capacitor whose capacitance varies with the amount of water inside.



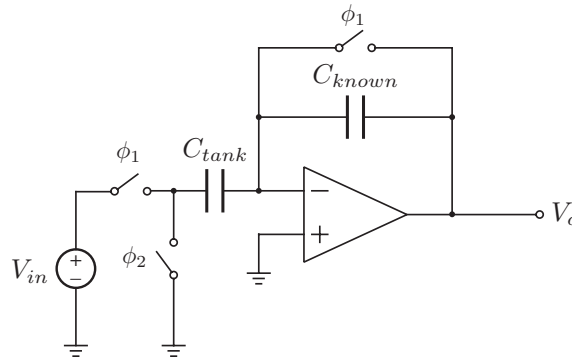
The width and length of the tank are both  $w$  (i.e. the base is square) and the height of the tank is  $h_{tot}$ .

- What is the capacitance between terminals  $a$  and  $b$  when the tank is full? What about when it is empty?  
Note: the permittivity of air is  $\epsilon$ , and the permittivity of rainwater is  $81\epsilon$ .
- Suppose the height of the water in the tank is  $h_{H_2O}$ . Modeling the tank as a pair of capacitors in parallel, find the total capacitance between the two plates. Call this capacitance  $C_{tank}$ .
- After building this capacitor, the farmer consults the Internet to assist her with a capacitance measuring circuit. A random Anon recommends the following:



In this circuit,  $C_{tank}$  is the total tank capacitance that you calculated earlier.  $C_{known}$  is some fixed and known capacitor. Find the voltage  $V_o$  in phase  $\phi_2$  as a function of the height of the water. Note that in phase  $\phi_1$  all switches labeled  $\phi_1$  are closed and all switches labeled  $\phi_2$  are open. In phase  $\phi_2$ , all switches labeled  $\phi_1$  are open and all switches labeled  $\phi_2$  are closed. You should also assume that before any measurements are taken, the voltages across both  $C_{known}$  and  $C_{tank}$  are initialized to  $0V$ .

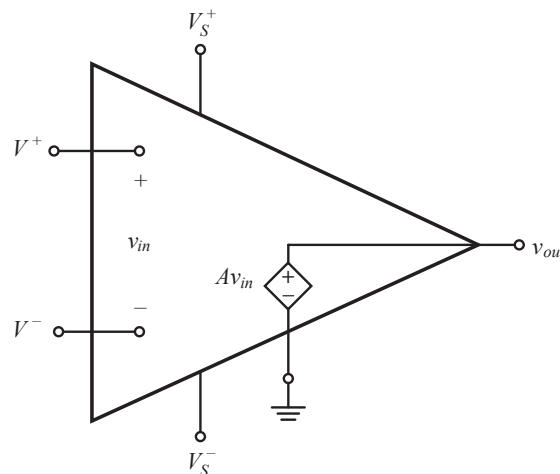
- (d) Use IPython (or any other tool or just do it by hand) to plot this voltage  $V_o$  as a function of the height of the water. Vary the tank from empty to full. Use values of  $V_{in} = 12V$ ,  $w = 0.5m$ ,  $h_{tot} = 1m$ , and  $\epsilon = 8.854 \times 10^{-12}F/m$ . This  $\epsilon$  is called the *permittivity of free space*. For  $C_{known}$  use a similar tank that is known to always be empty.
- (e) With the previous part, we were able to derive an expression for  $V_o$ . What does  $V_o$  represent? It's something we can measure! Our original goal was to determine what the height of the water in the tank without having to look inside it. Rewrite the last part to solve for  $h_{water}$ .
- (f) How about we perform a sanity check on our answer. What are the units of your result for  $V_o$  and for  $h_{water}$ ?
- (g) **(BONUS In-scope)** The farmer has become tired of solving the equation and wishes to generate a voltage proportional to the tank capacitance. A brief consultation with her daughter, yields the following circuit:



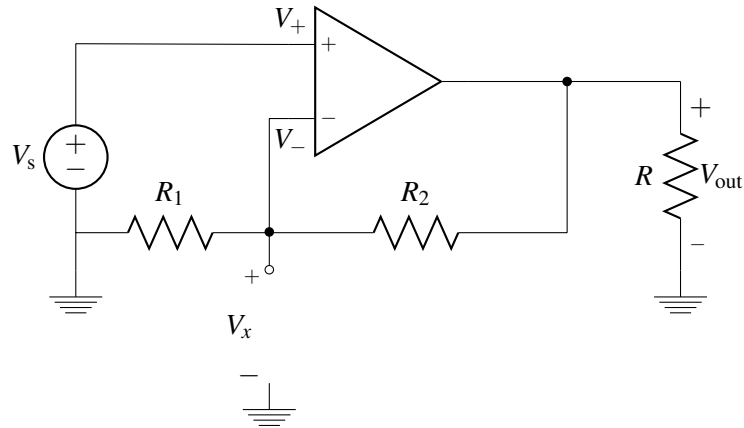
Calculate  $V_o$  as a function of  $h_{H_2O}$  in phase  $\phi_2$ . Use the Golden Rules. (*Hint: think about what must happen to the charge on the capacitor  $C_{tank}$  in phase  $\phi_2$ . Where does that charge have to go?*)

## 5. Op-Amp Golden Rules

In this question we are going to show that the golden rules for op-amps hold by analyzing equivalent circuits and then taking the limit as the open-loop gain approaches infinity. Below is a picture of the equivalent model of an op-amp we are using for this question.



- (a) Now consider the circuit below. Draw an equivalent circuit using the op-amp model shown above and calculate  $V_{out}$  and  $V_x$  in terms of  $A$ ,  $V_s$ ,  $R_1$ ,  $R_2$  and  $R$ . Is the magnitude of  $V_x$  larger or smaller than the magnitude of  $V_s$ ? Do these values depend on  $R$ ?



- (b) Using your solution to part (a), calculate  $V_{out}$  and  $V_x$  in the limit as  $A \rightarrow \infty$ . Do you get the same answers if you apply the golden rules ( $V_+ = V_-$  when there is negative feedback)?

## 6. Cool For The Summer

You and a friend want to make a box that helps control an air-conditioning unit. You both have dials that emit a voltage: 0 means you want to leave the temperature as it is. Negative voltages mean that you want to reduce the temperature. (It's hot so we will assume that you never want to increase the temperature — so, we're not talking about a Berkeley summer...)

Your air-conditioning unit, however, responds to positive voltages. The higher the voltage, the more strongly it runs. At zero, it is off. (If it helps, think of this air-conditioning unit as a heat pump. If you run it with negative voltage, it pumps heat in the opposite direction — from outside to inside. If positive voltage, it pumps heat from inside to outside.)

So you need a box that is an inverting summer — it outputs a weighted sum of two voltages where the weights are both negative. (Weighted because each of you has your own subjective sense for how much to turn the dial down and you need to compensate for that.)

This problem walks you through this using an op-amp.

- As a first step, create a general inverting amplifier and find the voltage gain.
- Now add a second input to the amplifier from above. Find the overall voltage gain as a function of the two input voltages.
- Let's suppose you wanted to have the overall voltage gain be  $V_{out} = -(\frac{1}{4}V_{S1} + 2V_{S2})$  where  $V_{S1}$  and  $V_{S2}$  represent the input voltages from you and a friend. Select resistors values such that this is the overall voltage gain.
- Now suppose you have another AC unit you want to add to the same room. This unit however, functions opposite to the already existing unit, it responds to negative voltages. You want to run both units at the same time. Add another op-amp to this circuit to create an output for the second AC unit.

- 7. Your Own Problem** Write your own problem related to this week's material and solve it. You may still work in groups to brainstorm problems, but each student should submit a unique problem. What is the problem? How to formulate it? How to solve it? What is the solution?