1

Consider a sample of material of cross section A where the conduction electron concentration varies in the x-direction, but is constant over each cross-section. Let $n(x)$ denote the density of conduction electrons (measured in m^{-3}) at location x. There will be a net movement of electrons in order to even out the imbalance in density; this process is called *diffusion*. Because electrons are charged particles, this diffusion will result in a current, called the diffusion current. We will normalize this to a per-unit-cross-sectional-area basis and talk about it via the *diffusion current density*, denoted J_n^{diff}, measured in amps/m^2. One can figure out a formula for the diffusion current density by visualizing the movement of electrons during their free motion between collisions after breaking the x-direction up into little intervals of length dx. The rate of flow of electrons from the left to the right across the cross-section at x should be roughly proportional to $n(x - \frac{dx}{2})$, while that from the right to the left should be roughly proportional to $n(x + \frac{dx}{2})$. The proportionality constant should be the same because the rate of flow is proportional to the number of particles flowing, whatever may be the details of how the particles move. Thus the diffusion current density should be proportional to the *gradient* of the conduction electron density. This is called *Fick's law*. We write

$$J_n^{diff}(x) = qD_n \frac{dn}{dx}$$

where D_n is called the *diffusion coefficient* for electrons, measured in m^2/sec. Note that an increasing conduction electron density (positive $\frac{dn}{dx}$) leads to a net flow of electrons to the left which represents a positive current to the right, so the sign in this equation is correct. Likewise we will have

$$J_p^{diff}(x) = -qD_p \frac{dp}{dx}$$

where $p(x)$ denotes the density of holes, which we think of as varying only in the x-direction, being constant over each cross-section, J_p^{diff} is the hole current density, measured in amps/m^2, and D_p is the diffusion coefficient for holes, measured in m^2/sec.

Typical values at room temperature are $D_n = 25$ cm^2/sec and $D_p = 10$ cm^2/sec. This is also consistent with the Einstein relation, see below.

2

The *Einstein relation* or *Einstein-Smoluchowski relation* gives a fundamental connection between the diffusion coefficient and the mobility for both electrons and holes. We have

$$D_n = \frac{kT}{q}\mu_n \text{ and } D_p = \frac{kT}{q}\mu_p.$$

In these equations recall that k and q are fundamental constants of nature (k is the Boltzmann constant and q is the charge of the proton), while T denotes that absolute temperature. As a sanity check, you should verify that the units work out in these formulas ($\frac{kT}{q}$ has the units of Volts). This is an example of a *fluctuation-dissipation theorem*.

The Einstein relation was derived in 1905 by Einstein in his study of *Brownian motion* (the phenomenon, observed in the 1780's by the botanist Robert Brown, that very light particles such as pollen, when suspended in a liquid, appear to exhibit random motions: this is now understood to be the result of the random bombardment of the suspended particles by the molecules of the fluid; at the time it was unclear whether pollen was in fact alive). The original derivation of Einstein was in a mechanical context, but the principles he used are much more general in scope. The Einstein relation, as applied to electrons (or holes), comes from applying the principles of the kinetic theory of gases to an "electron gas". We now derive the Einstein relation for electrons (and holes). The following section is optional: you can skip reading it if you want.

3 Optional: you may skip this section if you prefer.

We consider the case of electrons; a parallel discussion may be carried out for holes. Consider a section of the slab of material of length dx in the x-direction, with the cross-sectional area A. We think of dx as being very small. Recall that n denotes the density of conduction electrons per unit volume. The total number of conduction electrons in the slab is then roughly $nAdx$.

We think of the conduction electrons as forming an "electron gas". This electron gas exerts a "pressure", according to the kinetic theory of gases. We will use the notation π for this pressure (since p has already been used up for the density of holes). Pressure is measured as force per unit area, as usual. Electrons are fermions, subject to the Pauli exclusion principle. Thus the electron gas is a "Fermi gas" and as such may be expected to behave differently from a classical ideal gas (for instance a Fermi gas has nonzero pressure even at absolute zero, because of the exclusion principle). However, we are only interested in discussing the conduction electrons. These are sufficiently energetic that we may think of them as forming a classical ideal gas.

Thinking of the conduction electron gas as an ideal gas, from the kinetic theory of gases we have the equation of state

$$\pi(Adx) = (nAdx)kT.$$

Here Adx is the volume of the slab and $nAdx$ is the number of electrons in the slab, so this is the analog of the equation $PV = NkT$ we saw a couple of lectures ago when we were discussing the ideal gas law. We thus get

$$\pi = nkT.$$

Now suppose the electron density is a function of x. Then the "pressure" is also a function of x, and this equation would read

$$\pi(x) = n(x)kT. \tag{1}$$

If the electron density is a function of x, the gradient in the pressure of the electron gas must manifest itself as a force on each electron (because the pressure on the electron from the left is different from that on it from the right). We denote this force per electron by $f(x)$ at location x. The net force from the left on the electrons in the slab is $\pi(x - \frac{dx}{2})A$, while the net force from the right is $\pi(x + \frac{dx}{2})A$. Since the total number of electrons in the slab is $n(x)Adx$, the force to the right per electron is

$$f(x) = \frac{\pi(x - \frac{dx}{2})A - \pi(x + \frac{dx}{2})A}{n(x)Adx}$$

3

This yields the equation

$$\frac{d\pi}{dx} = -n(x)f(x).$$

Electrons react to this force as they would to the force due to an electric field. Namely, because of collisions with the ions in the lattice, impurities, and crystal defects, the effect of this force is to set up a drift velocity, whose value is characterized by the mobility, as in the previous lecture. The force per electron to the right at location x, i.e. $f(x)$, has an effect equivalent to an electric field per electron to the left of $\frac{f(x)}{q}$ (recall that electrons move in a direction opposite to that of the electric field), so it results in a drift velocity $v(x)$ (note that this also depends on x) of electrons to the right given by

$$v(x) = \mu_n \frac{f(x)}{q},$$

where μ_n denotes the mobility of electrons. Combining this with the preceding equation, we get

$$\frac{d\pi}{dx} = -\frac{qn(x)v(x)}{\mu_n} \tag{2}$$

On the other hand, we have Fick's law, which tells us that the current due to electrons moving to the right across the cross-sectional area of A at location x during a very small interval of time dt can be written as

$$qD_n \frac{dn}{dx} A dt$$

where D_n is the diffusion coefficient. This number must, however, also equal

$$-qn(x)Av(x)dt$$

because $v(x)dt$ is the net distance moved by electrons with drift velocity $v(x)$ in a time interval dt (note that "drift velocity" is a notion referring to overall movement by the electron gas at any location and not to the motion of individual electrons; the latter also has a substantial random component). This gives the equation

$$-D_n \frac{dn}{dx} = n(x)v(x) \tag{3}$$

Combining equations (1), (2) and (3) proves the Einstein relation, as you can easily verify.

4

4

Consider a slab of silicon of cross section A, with all variations being in the x-direction. Let $n(x)$ denote the density of conduction electrons and $p(x)$ the density of holes in the valence band. In thermal equilibrium there should be no net flow of current across any cross-section. For electrons, this gives the equation

$$qn(x)\mu_n E(x) + qD_n \frac{dn}{dx} = 0$$

where we have written $E(x)$ to emphasize that we think of the electric field as varying with x (this electric field includes the possibility of an externally applied field, but also the electric fields that result from movement of the free electrons and holes, as e.g. near a p-n junction, which is the case of most interest to us). This can be rearranged to give

$$\frac{dn}{dx} = -\frac{\mu_n}{D_n}n(x)E(x) = -\frac{q}{kT}n(x)E(x)$$

where the second step uses Einstein's relation. Writing the electric field as the gradient of a potential

$$E(x) = -\frac{d\phi}{dx}$$

(where the potential is only defined up to the choice of an arbitrary reference potential) and solving, we get for any two points x and x_0, the equation

$$\phi(x) - \phi(x_0) = V_T \ln \frac{n(x)}{n(x_0)}$$

where $V_T = \frac{kT}{q}$ is the thermal voltage (roughly 26 millivolts at room temperature). It is conventional to choose the potential reference by setting $\phi(x_0) = 0$ at a hypothetical point x_0 where $n(x_0)$ equals the intrinsic carrier concentration n_i (which is roughly $10^{10} \ cm^{-3}$ for silicon). With this choice we get the equation

$$\phi(x) = V_T \ln \frac{n(x)}{n_i} \tag{4}$$

which is used in your homework set.

By the mass action law, we have $n(x)p(x) = n_i^2$. Substituting this, into the preceding equation would give

$$\phi(x) = V_T \ln \frac{p(x)}{n_i}. \tag{5}$$

This could also have been arrived at directly by starting with holes and going through the steps in the preceding paragraph.

5

You can now read through the supplementary reader starting with Section 3.2 and going through to the end of Section 3.3.2. We will discuss the rest of Chapter 3 of the supplementary reader (which deals with biasing of p-n junction diodes) at the beginning of the next lecture. There is one important point to keep in mind that connects the formulas you see in the supplementary reader for the potential and the formulas we need, the way the potential has been defined in this class. The function called $\phi_0(x)$ is the potential when we choose as reference that the potential in the bulk p-type material is 0, i.e. that $\phi_0(-x_{p0}) = 0$ (see the line just about equation (15) of the supplementary notes; here $-x_{p0}$ denotes the edge of the depletion region in the p-type region, under the depletion approximation). However, the potential, as defined in class, is the function $\phi(x)$ which satisfies both the formulas (4) and (5). Thus we have

$$\phi(-x_{p0}) = -V_T \ln \frac{N_a}{n_i}$$

where N_a denotes the doping density (of acceptors) in the p-type region and n_i the intrinsic carrier concentration, and

$$\phi(-x_{n0}) = -V_T \ln \frac{N_d}{n_i}$$

where N_d denotes the doping density (of donors) in the n-type region. In other words, we have

$$\phi(x) = \phi_0(x) - V_T \ln \frac{N_a}{n_i}.$$

Because the p-n junction as a whole is electrically neutral, we must have, under the depletion approximation, the equation

$$qN_ax_{p0} = qN_dx_{n0} \tag{6}$$

(this is equation (12) of the supplementary notes). In the supplementary notes not enough information has been given to allow you to figure out the actual values of x_{n0} and x_{p0}. However, we do have enough information to do this, because of the equations (4) and (5). From these equations we know that the total rise in potential across the depletion region must equal

$$\phi_B = V_T \ln \frac{N_aN_d}{n_i^2}$$

(where this equation defines the notation ϕ_B, which is called the *built-in potential* at the junction). Given the equation for $\phi(x)$ (or equivalently of $\phi_0(x)$) derived in the supplementary notes, the requirement that the total rise in potential across the junction equals ϕ_B allows us to determine $x_{n0} + x_{p0}$ and from this we can determine both x_{n0} and x_{p0} by using equation (6). The details of this algebra are easy, so you can do them yourself you wish. The answers we get are

$$x_{n0} = \sqrt{\frac{2\epsilon_s\phi_B}{qN_d}\frac{N_a}{N_a + N_d}}$$

and

$$x_{p0} = \sqrt{\frac{2\epsilon_s\phi_B}{qN_a}\frac{N_d}{N_a + N_d}},$$

where ϵ_s denotes the permittivity of silicon given roughly by $\epsilon_s = 11.7\epsilon_0$, where ϵ_0 denotes the permittivity of vacuum. You can take $\epsilon_0 = 8.85 \times 10^{-12}$ Farads/m, so you can take $\epsilon_s = 1.035 \times 10^{-10}$ Farads/m.

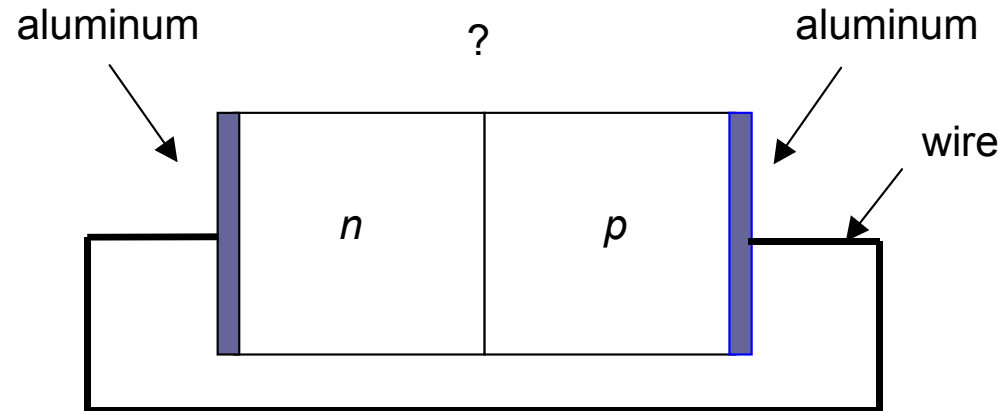All the formulas that you need to do Homework 9 are now available.

Junctions of n- and p-type Regions

p-n junctions form the essential basis of all semiconductor devices.

Therefore, understanding the p-n junction is important.

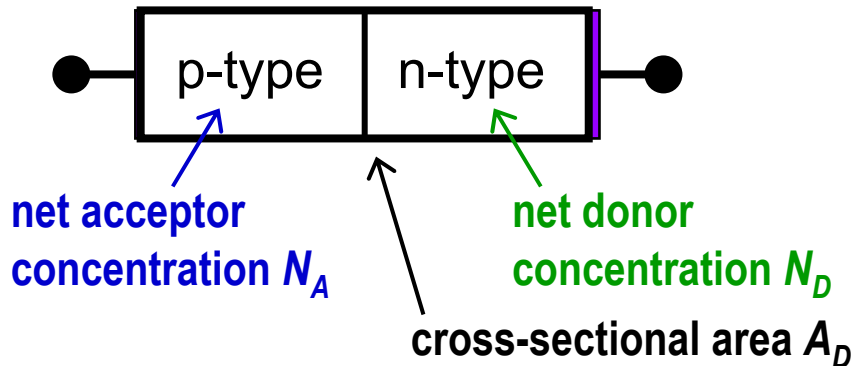What happens to the electrons and holes when

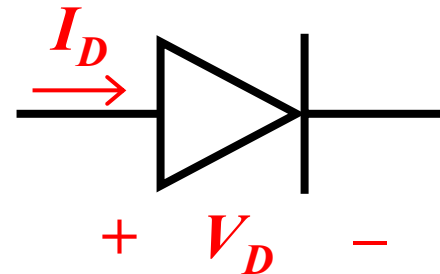**n and p regions are brought into contact :**

aluminum   ?   aluminum

wire

n p

*Note that the textbook has a very good explanation.

The *pn* Junction Diode

Schematic diagram

p-type | n-type

net acceptor concentration N_A

net donor concentration N_D

cross-sectional area A_D

Circuit symbol

I_D

$+$ V_D $-$

Physical structure:
(an example)

$+$ I_D

metal

SiO$_2$ SiO$_2$

p-type Si

V_D

n-type Si

$-$ metal

For simplicity, assume that the doping profile changes abruptly at the junction.
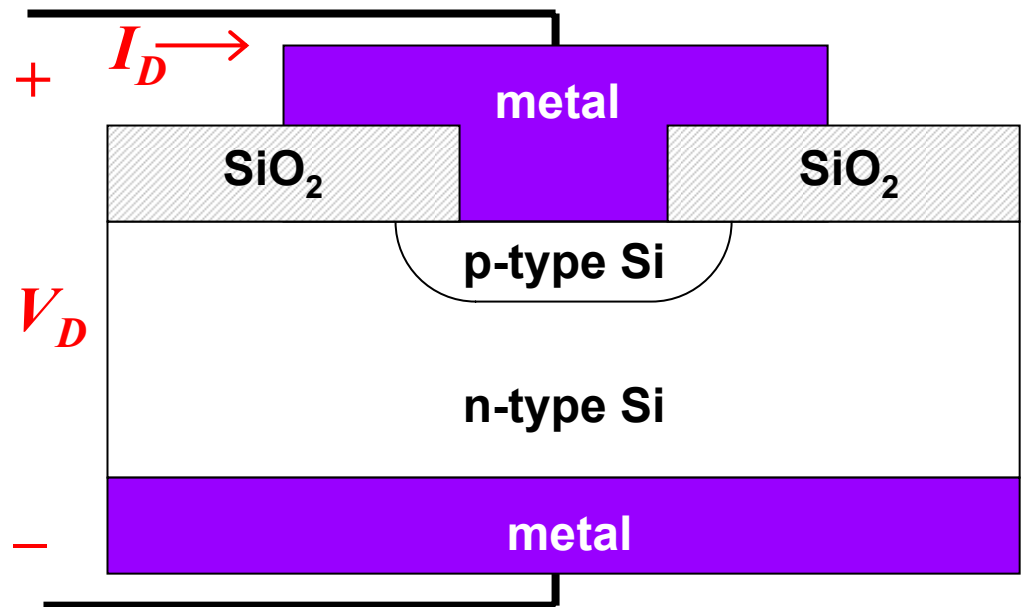
Depletion Region *Approximation*

- When the junction is first formed, mobile carriers **diffuse** across the junction (due to the concentration gradients)

  - Holes diffuse from the p side to the n side, leaving behind negatively charged immobile acceptor ions

  - Electrons diffuse from the n side to the p side, leaving behind positively charged immobile donor ions

acceptor ions   donor ions
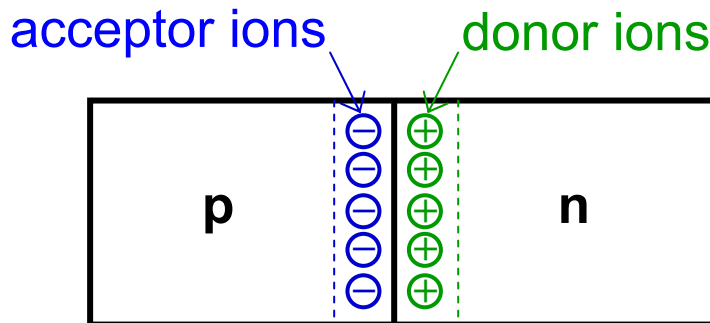
→**A region depleted of mobile carriers is formed at the junction.**

- The space charge due to immobile ions in the depletion region establishes an electric field that opposes carrier diffusion.

Charge Density Distribution

Charge is stored in the depletion region.

acceptor ions   donor ions

p   n

quasi-neutral p region   depletion region   quasi-neutral n region

charge density (C/cm^3)

distance