POWER AND ENERGY IN ELECTRIC CIRCUITS

Energy: Volts X Coulombs (e.g. raise one coulomb up through one volt in potential and you have done 1 Joule of work (i.e. delivered one Joule of energy).

Power: Transfer of energy per unit time (Joules per second = Watts)

Concept: in falling through a positive potential drop V, a positive charge q dissipates energy equal to qV.

- potential energy change = qV for each charge q
- Rate is given by # charges/sec

Power = P = V (dq/dt) = VI

So Power = voltage X current

POWER IN ELECTRIC CIRCUITS

$P = V \times I$ Volt $\times$ Amps = Volts $\times$ Coulombs/s = Joules/s = Watts

Circuit elements can *absorb* or *release* power (I.e., from or to the rest of the circuit); power can be a function of time.

How to keep the signs straight for absorbing and releasing power?

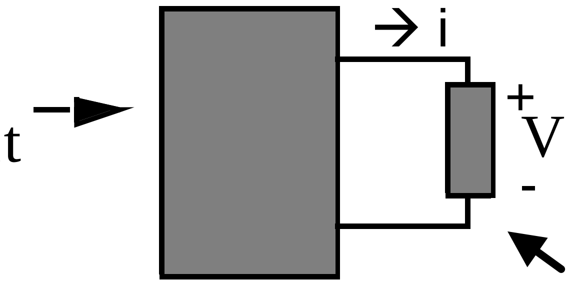
+ Power $\equiv$ absorbed into element

– Power $\equiv$ delivered from element

"ASSOCIATED REFERENCE DIRECTIONS"

Associated reference directions refers to defining the current *through* a circuit element as positive when entering the terminal associated with the + reference for voltage

This box represents the rest of the circuit



Circuit element

For positive current and positive voltage, positive charge "falls down" a potential "drop" in moving through the circuit element: it *absorbs* power

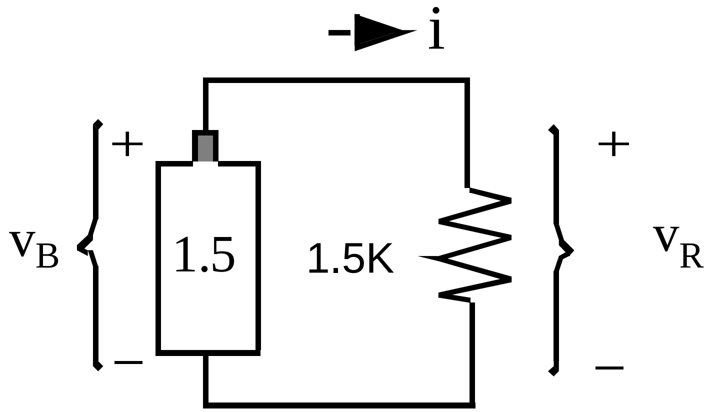
- P = VI > 0 corresponds to the element absorbing power {ONLY FOR ASSOCIATED REFERENCE DIRECTIONS}

How can a circuit element absorb power?

By converting electrical energy into heat (resistors in toasters); light (light bulbs); acoustic energy (speakers); by storing energy (charging a battery).

Negative power $\Rightarrow$ releasing power to rest of circuit

"ASSOCIATED REFENCE DIRECTIONS" (cont.)



We know **i** = +1 mA

A) For Resistor: $P = i\, v_R$
(because we are using "associated reference directions")

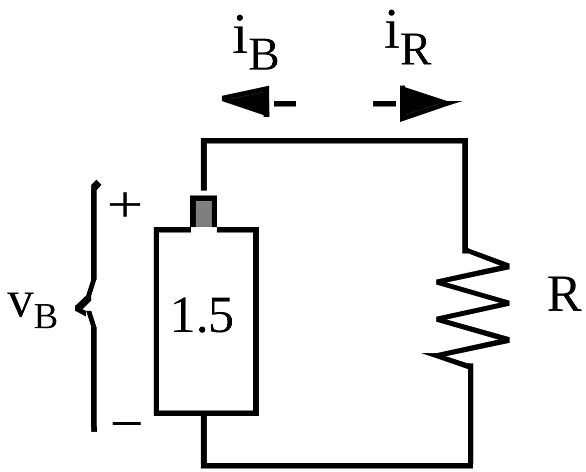
Hence, $P = i\, v_R = +1.5\text{mW}$
(absorbed)

Note for resistor (only) $P=i^2R$ or V^2/R (substitution of ohms law)

B) Note that for the battery, current **i** is **opposite** to associated

So, $P = (-i)v_B = -1.5\,\text{mW}$
(delivered out of battery)

"ASSOCIATED REFENCE DIRECTIONS" (cont.)



Again, i_R = 1mA

therefore i_B = -1mA

Positive sign because of associated directions

A) Resistor: P = i_B v_B = +1.5mW

Positive sign thus power is absorbed

B) Battery: i_B and v_B are associated, therefore P= i_B v_B .
Thus $P = 1.5 \times (-1 \times 10^{-3}) = -1.5\,\mathrm{mW}$

Negative sign thus power is delivered

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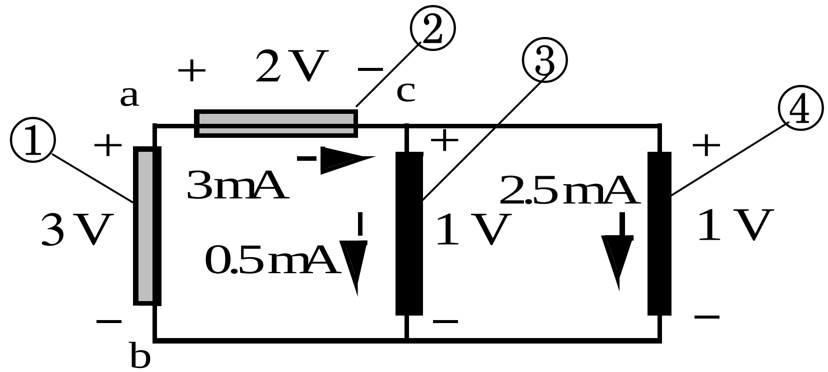
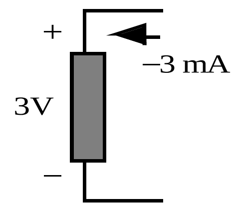
EXAMPLES OF CALCULATING POWER

Find the power absorbed by each element



Element ①: flip current direction:



$$P_1 = 3\text{V}(-3\,\text{mA}) = -9\,\text{mW}$$

Element ②: $P_2 = 2\text{V}(3\,\text{mA}) = 6\,\text{mW}$

Element ③: $P_3 = 1\text{V}(0.5\,\text{mA}) = 0.5\,\text{mW}$

Element ④: $P_4 = 1\text{V}(2.5\,\text{mA}) = 2.5\,\text{mW}$

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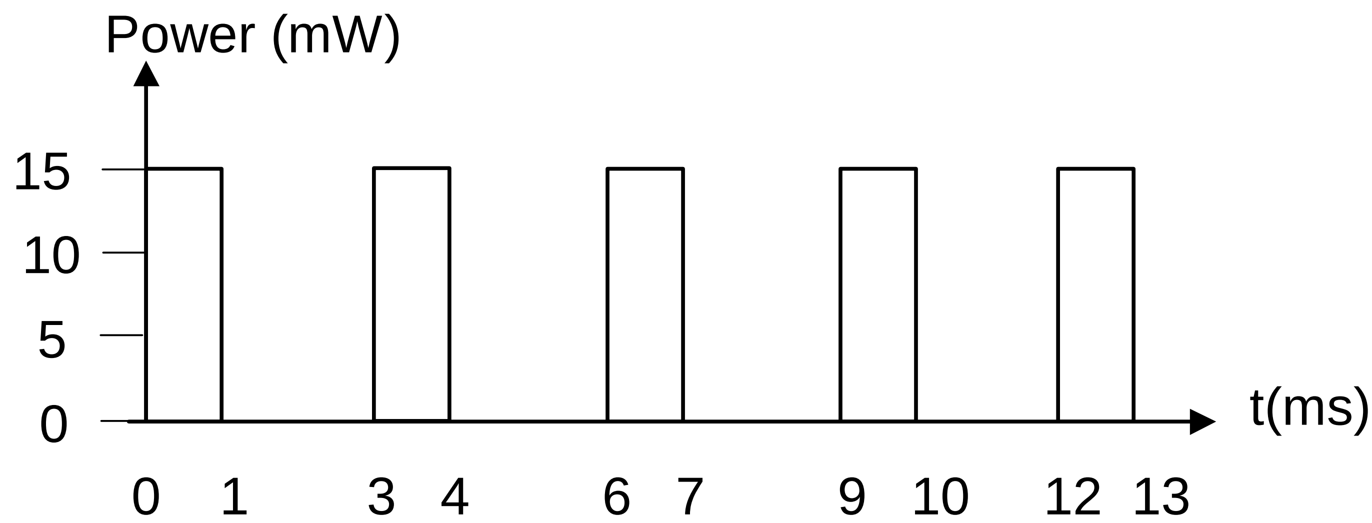
EXAMPLES OF CALCULATING POWER in non-DC situations

- Time-averaged power - just average P(t) This can be viewed as plotting Power versus time, computing the area under the curve over some interval, and dividing by the interval.

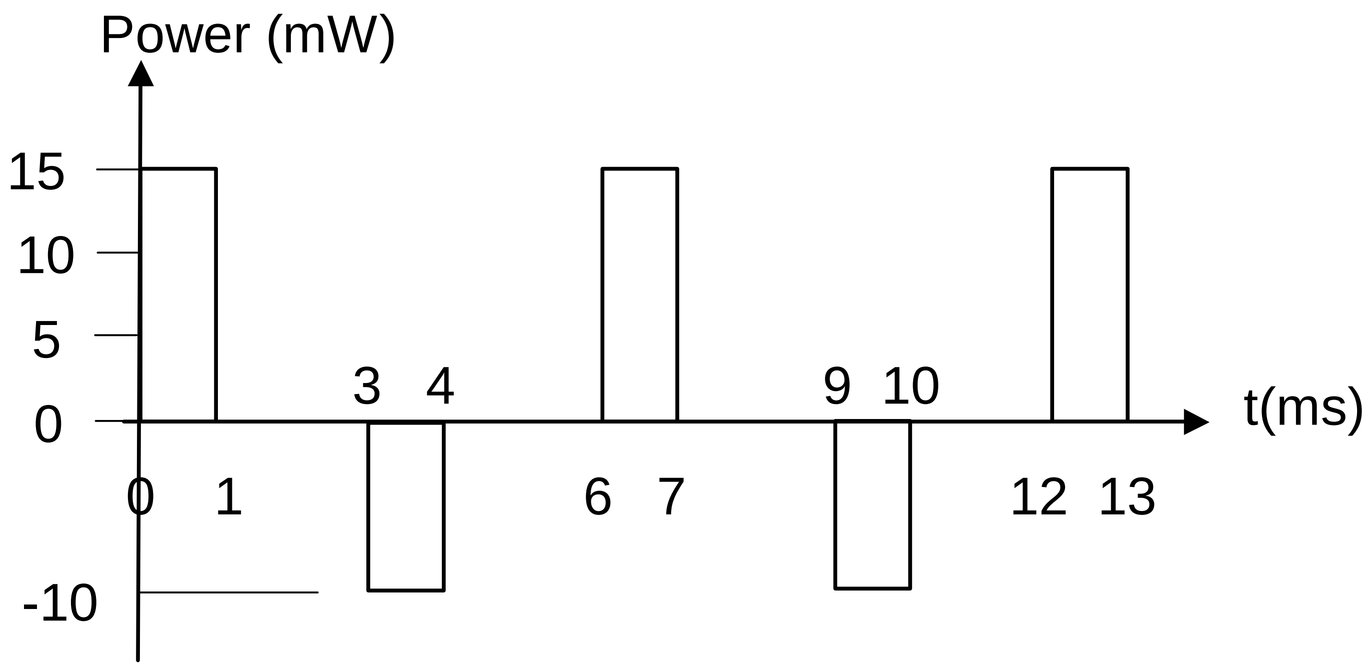


Lets average over the 3ms period of the waveform:

Clearly the average power is 15/3 mW or 5mW.

EXAMPLES OF CALCULATING POWER in non-DC situations

- Note that P(t) can be positive or negative (part of the time absorbing power, part of the time producing power)



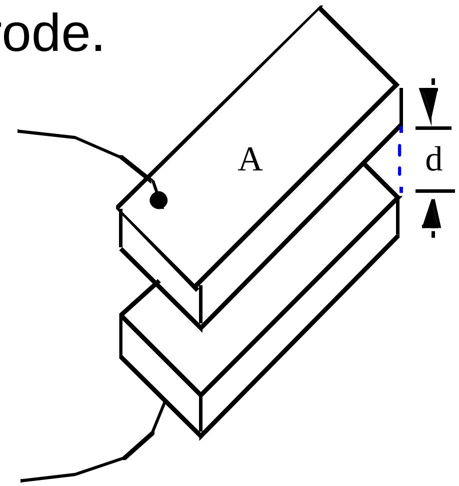
Lets average over a 6ms interval (that is the period)

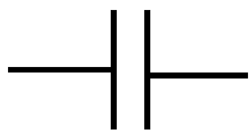
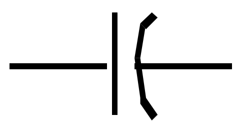
Now the average power is (15 –10)/6 mW or 5/6 mW.

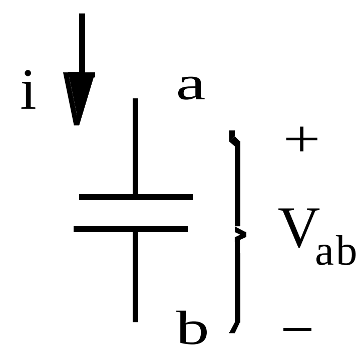
CAPACITOR

Any two conductors a and b separated by an insulator with a difference in voltage V_{ab} will have an equal and opposite charge on their surfaces whose value is given by $Q = CV_{ab}$, where C is the **capacitance** of the structure, and the + charge is on the more positive electrode.

We learned about the *parallel-plate capacitor* in physics. If the area of the plate is A, the separation d, and the *dielectric constant* of the insulator is ε, the capacitance equals $C = A\,\varepsilon/d$.



Symbol  or 



Constitutive relationship: $Q = C\,(V_a - V_b)$.

(Q is positive on plate a if $V_a > V_b$)

But $i = \frac{dQ_a}{dt}$ so $\boxed{i = C\frac{dv}{dt}}$

where we use the *associated reference directions*.

ENERGY STORED IN A CAPACITOR

You might think the energy (in Joules) is QV, which has the dimension of joules. But during charging the average voltage was only half the final value of V.

Thus, energy is $\frac{1}{2}\boldsymbol{QV} = \boxed{\frac{1}{2}\boldsymbol{CV}^2}$.

(To see this clearly, plot voltage and charge versus time for a constant current into a capacitor and think about the energy during some incremental charging interval V(t) dQ and then reckon what the average voltage is during the charging up to the final capacitor voltage V).

ENERGY STORED IN A CAPACITOR EXAMPLES

Examples to be worked in class

1) Battery charging a capacitor to voltage V_B, Find energy dissipated in resistor, energy delivered by battery, and energy stored in capacitor.

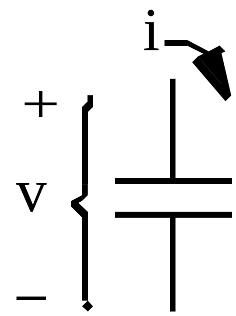
Answers: $1/2CV^2$, CV^2, $1/2CV^2$

2) Capacitor discharging into a resistor. Find the energy lost from the capacitor and the energy dissipated in the resistor.

Answer: $1/2CV^2$, $1/2CV^2$

ENERGY STORED IN A CAPACITOR (cont.)
(Not to be covered, just for fun)

More rigorous derivation: During charging, the power flow is $v \cdot i$ into the capacitor, where i is into + terminal. We integrate the power from t = 0 (v = 0) to t = end (v = V). The integrated power is the energy



$$E = \int_{0}^{t=\text{end}} v \cdot i \, dt = \int_{0}^{\text{end}} v \frac{dq}{dt} dt = \int_{v=0}^{v=V} v \, dq$$

but dq = C dv. (We are using small q instead of Q to remind us that it is time varying . Most texts use Q.)

$$E = \int_{v=0}^{v=V} Cv \, dv = \frac{1}{2} CV^2$$

INDUCTORS

Inductors are the **dual** of capacitors – they store energy in magnetic fields that are proportional to current.

Capacitor	**Inductor**
$i = C\frac{dV}{dt}$	$v = L\frac{di}{dt}$
$E = \frac{1}{2}CV^2$	$E = \frac{1}{2}LI^2$

SWITCHING PROPERTIS OF L, C

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Rule: The voltage across a capacitor must be continuous and differentiable

Basis: The energy cannot "jump" (else infinite energy flow); and of course the current cannot be infinite, so dV/dt must be finite.

Just as capacitors demand v be continuous (no jumps in V), inductors demand i be continuous (no jumps in i). Reason? In both cases the continuity follows from non-infinite, i.e., finite, power flow.

Capacitor	**Inductor**
v is continous	i is continous
I can jump	V can jump
Do not short circuit a charged capacitor (produces ∞ current)	Do not open an inductor with current flowing (produces ∞ voltage)